

# CS 110

# Computer Architecture

# Datapath

Instructors:

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Course website: [https://toast-](https://toast-lab.sist.shanghaitech.edu.cn/courses/CS110@ShanghaiTech/Spring-2025/index.html)

**School of Information Science and Technology (SIST)**

**ShanghaiTech University**

2024/3/25

# Administratives

- Lab 5 available, please prepare in advance, to check this week!  
Lab 6 released!
- HW 3 available, ddl April 1st, start early!
- Proj 1.1 ddl TODAY, Mar. 27th!
- Proj 1.2 will be released.
- Discussion this week on digital circuits.

# Mid-term I

- Midterm I
  - April 10th **8:00 am - 10:00 am**
    - We start sharp at 8:00 am!
    - Arrive **7:45 am** to check-in (Venue: TBD on your egate system; Seat: TBD on-site)
    - Arrive later then **8:30 am** will get 0 mark.
- Contents:
  - Everything till April 8th lecture
  - Switch cell phones **off!!!** (not silent mode)
    - Put them in your bags.
  - Bags in the front. On the table: nothing but pen, exam paper, 1 drink, 1 snack, **your student ID card** and **your cheat sheet!**

# Mid-term I requirements

- You can bring a cheatsheet (**handwritten only**). **1-page A4, double-sided** (2-page for the mid-term II and 3-page for the final). Put it on your desk at exam. Cheatsheet that does not apply to the rules would be taken away.
- [Greencard](#) shown on the course website is provided with the exam paper.
- No other electronic devices are allowed!
  - No ear plugs, music, smartwatch, calculator, computer...
- Anybody touching any electronic device will **FAIL** the course!
- Anybody found cheating (copy your neighbors answers, additional material, ...) will **FAIL** the course!





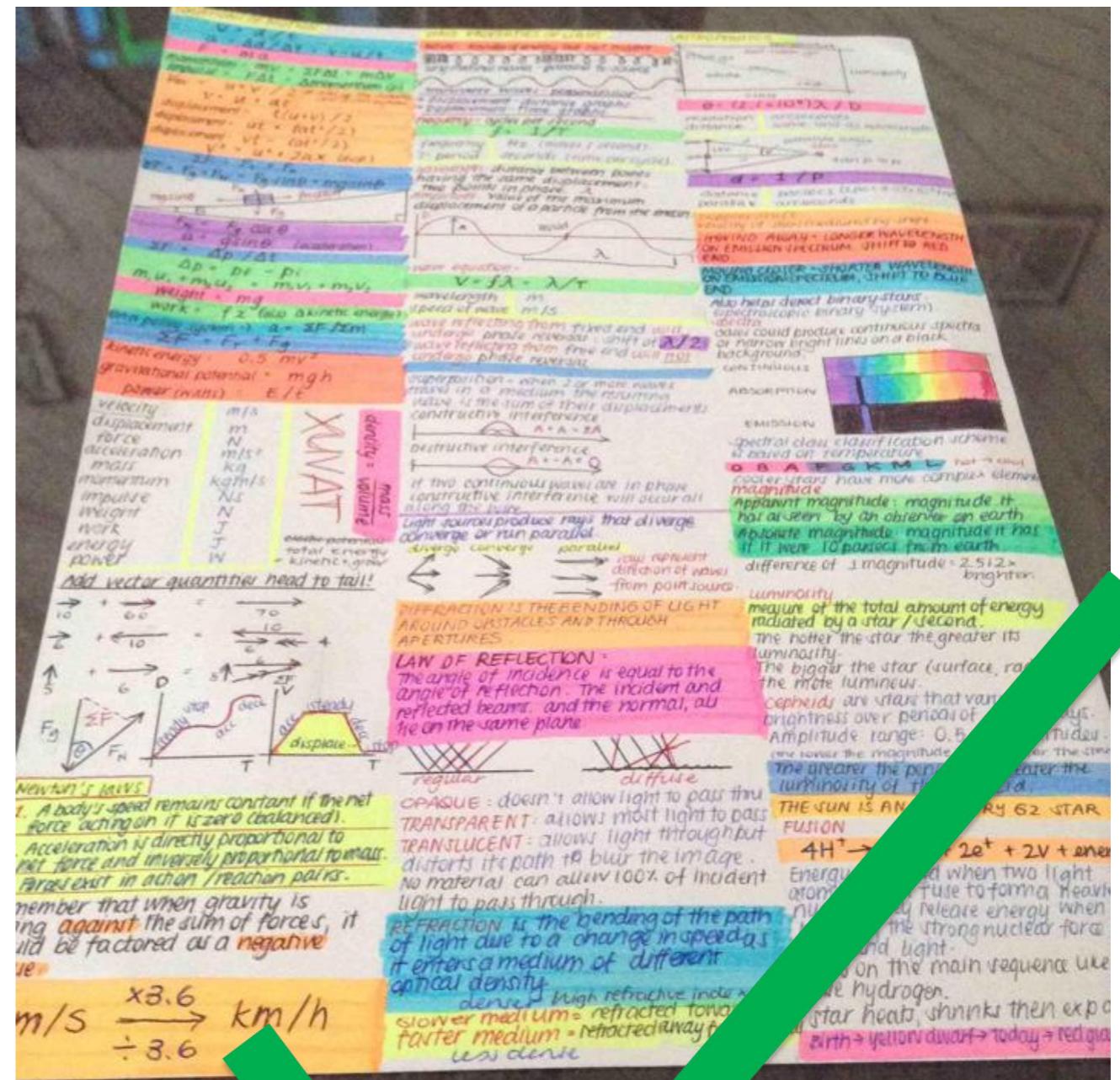


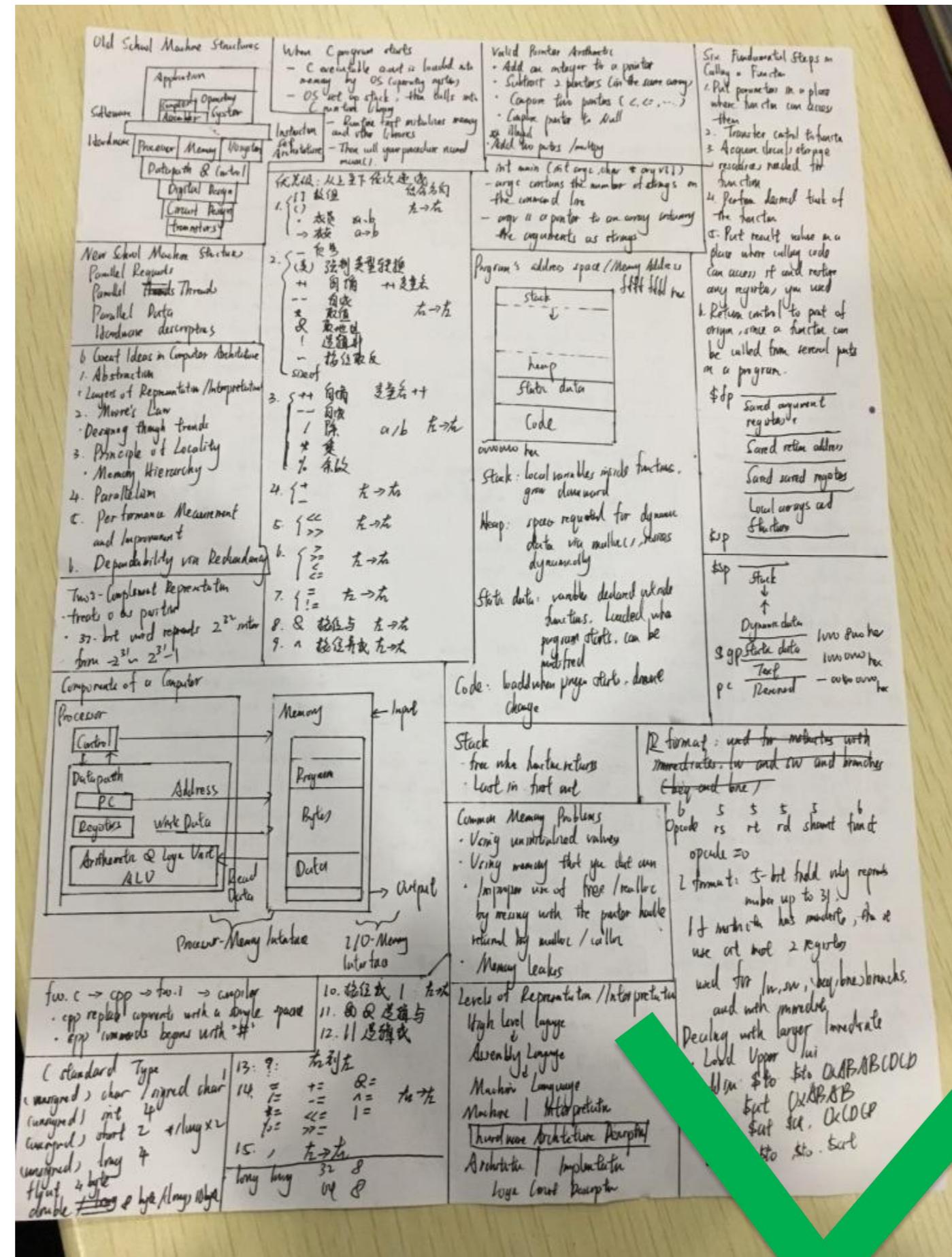


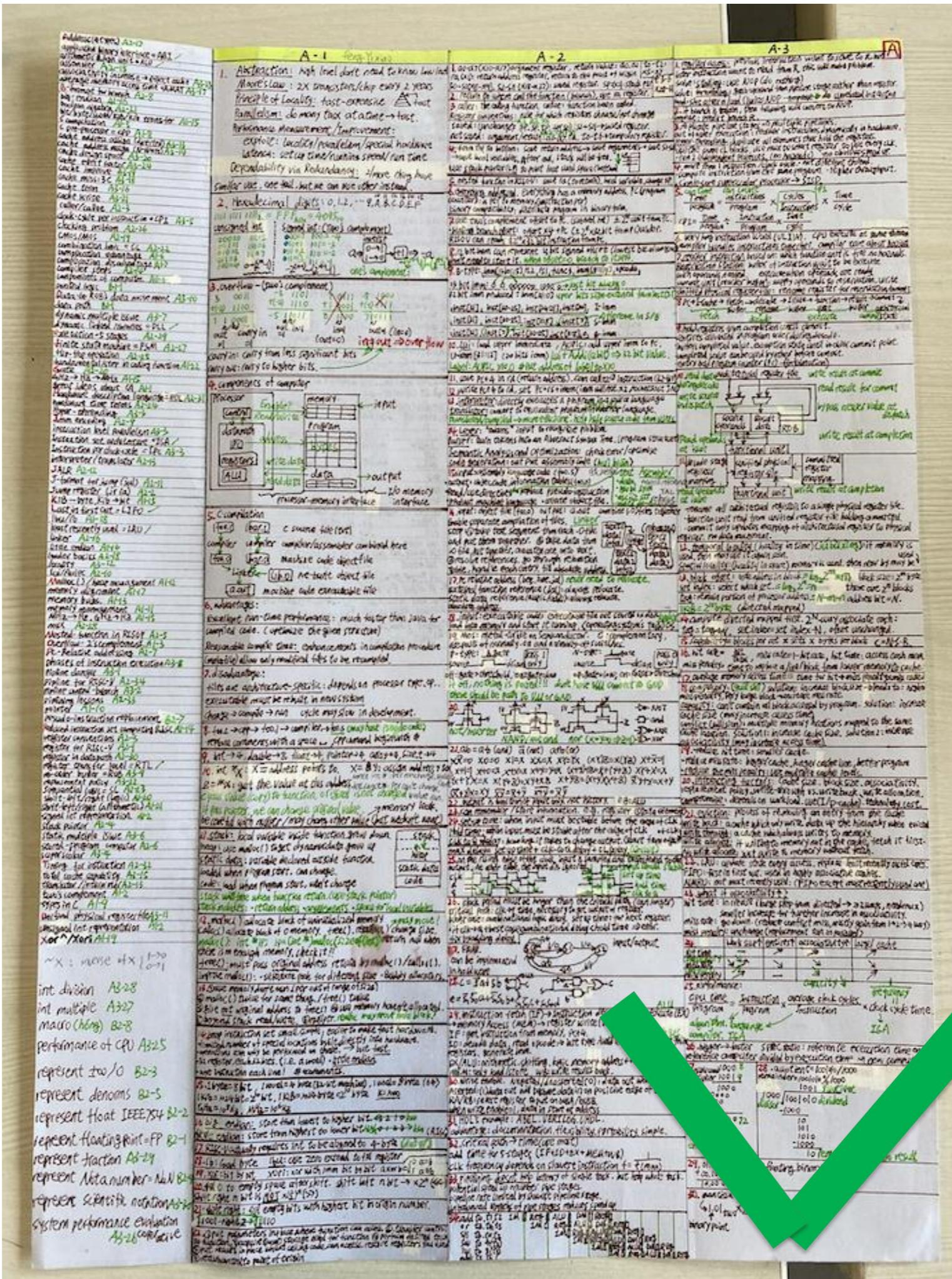
# Cheat Sheet

- 1 A4 Cheat Sheet allowed (double sided)
  - Midterm II: 2 pages
  - Final: 3 pages
- Rules:
  - *Hand-written* – not printed/photocopied!
  - Your name in pinyin on the top!
  - Cheat Sheets not complying to this rule will be confiscated!

FUNCTIONS OF SEVERAL VARIABLES		$z = f(x, y)$ , $w = f(x, y, z)$	DOMAIN: ALLOWED $(x, y)$ , $(x, y, z)$ RANGES: $z, w$ 's
LEVEL CURVES	$z = f(x, y) = k = \text{CONST.}$	FUNCTION OF $n$ VARIABLES	$z = f(x_1, x_2, \dots, x_n)$ , $w = f(x_1, y, z)$
$z = f(x, y) = k = \text{CONST.}$	$\text{CONTOUR MAPS (2-D)}$	$z = f(x_1, x_2, \dots, x_n)$ , $w = f(x_1, y, z)$	$z = f(x_1, x_2, \dots, x_n)$ , $w = f(x_1, y, z)$
$w = f(x, y, z) = k = \text{CONST.}$	$\text{SURFACE LAYERS (3-D)}$	$z = f(x_1, x_2, \dots, x_n)$ , $w = f(x_1, y, z)$	$z = f(x_1, x_2, \dots, x_n)$ , $w = f(x_1, y, z)$
PARTIAL DERIVATIVES	$z = f(x, y)$ NOTATIONS	Derivatives w.r.t. one variable, while holding the other variables constant	$z = f(x_1, x_2, \dots, x_n)$ , $w = f(x_1, y, z)$
$f_x(x, y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial z}{\partial x}$	$f_y(x, y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial z}{\partial y}$	SAME HOLDS FOR FUNCTIONS OF MORE THAN TWO VARIABLES	$f_x(x_1, x_2, \dots, x_n) = \frac{\partial f}{\partial x_1} = \frac{\partial z}{\partial x_1}$ , $f_y(x_1, x_2, \dots, x_n) = \frac{\partial f}{\partial x_2} = \frac{\partial z}{\partial x_2}$ , $\dots$
SECOND PARTIAL DERIVATIVES	CLAIRAUT'S THEOREM		
$f_{xx} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2}{\partial x \partial x}$	IF $f_{xy}$ AND $f_{yx}$ ARE BOTH CONTINUOUS		
$f_{xy} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2}{\partial y \partial x}$	EQUATIONS OF TANGENT PLANES TO SURFACES		
$f_{yx} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2}{\partial x \partial y}$	$z = f(x, y) = f(x_1, y_1)$ EVALUATED AT A POINT		
$f_{yy} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2}{\partial y \partial y}$	$f_{xy} = f_{yx}$		
THE CHAIN RULE	SINGLE VARIABLE $y = f(u)$ , $u = g(t)$ , $z = f(g(t), h(t))$		
CASE 1	$z = f(x, y)$ , $x = g(t)$ , $y = h(t)$ i.e. $z = f(g(t), h(t))$		
	$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}$ or $w/z = f$ $\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t}$ SOME PARTIAL		
CASE 2	$z = f(x, y)$ , $x = g(s, t)$ , $y = h(s, t)$ i.e. $z = f(g(s, t), h(s, t))$		
	$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s}$ $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}$ THINGS APPEAR TO CANCEL SINCE BOTH $\frac{\partial x}{\partial s}$ AND $\frac{\partial y}{\partial s}$ DON'T DO THAT		
CHAIN RULE: GENERAL VERSION	$u = f(x_1, \dots, x_m)$ , $x_i = g_i(s, t)$ , $z = f(g_1(s, t), \dots, g_m(s, t))$		
	$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x_1} \frac{\partial x_1}{\partial s} + \frac{\partial z}{\partial x_2} \frac{\partial x_2}{\partial s} + \dots + \frac{\partial z}{\partial x_m} \frac{\partial x_m}{\partial s}$ for each $i = 1, 2, \dots, m$		
IMPLICIT DIFFERENTIATION	You can always solve for $y$ and diff. implicitly		
	$\frac{dy}{dx} = -\frac{F_x}{F_y}$		
	$\frac{\partial z}{\partial x} = -\frac{\partial F_x}{\partial x} = -\frac{F_x}{F_y}$		
	$\frac{\partial z}{\partial y} = -\frac{\partial F_x}{\partial y} = -\frac{F_x}{F_y}$		
	$F(x, y) = 0$ , $y = f(x)$ , $F_x, F_y \neq 0$		
	$F(x, y, z) = 0$ , $z = f(x, y)$ , $F_x, F_y, F_z \neq 0$		
TANGENT PLANE TO A LEVEL SURFACE	$\nabla F \perp$ TO TAN. PLANE		
	$F_x(x - x_0) + F_y(y - y_0) + F_z(z - z_0) = 0$		
	LEVEL SURFACE w/ $k = 0$		
SPECIAL CASE	$z = f(x, y)$ , $F(x, y, z) = f(x, y) - k = 0$		
	OLD DEFINITION		
	THEN $F_z = -1$ , $\nabla F = (F_x, F_y, -1)$ and TAN. PLANE $z - z_0 = f_x(x_0 - x_0) + f_y(y_0 - y_0)$		
MAXIMUM AND MINIMUM VALUES	$z = f(x, y)$		
	$f_x(a, b) = 0$ , $f_y(a, b) = 0$ , $\nabla f(a, b) = (0, 0)$ NECESSARY BUT NOT SUFFICIENT TO GUARANTEE A MAX. OR MIN.		
	SET $t^* = t^* = 0$ Solve for Critical Pts. (Always check $t^* = 0$ for min.) THEN APPLY THE 2ND DERIVATIVE TEST Find $f_{xx}$ , $f_{yy}$ , $f_{xy}$		
	$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - (f_{xy})^2$	$\begin{aligned} & \text{2} > 0, f_{xx} > 0 \text{ LOCAL MIN.} \\ & \text{D} > 0, f_{xx} < 0 \text{ LOCAL MAX.} \\ & \text{D} < 0 \text{ SADDLEPT. DO NOT TEST.} \end{aligned}$	
FINDING ABSOLUTE MAX. AND MINS. FOR $f$ ON A CLOSED, BOUNDED			
1. Find values of $f$ at the critical points of $f$ in $D$			
2. Find the extreme values of $f$ on the boundary of $D$			
3. The largest value from 1, 2, is the ABS. MAX., the smallest is ABS. MIN.			
MAXIMIZING AND MINIMIZING <sup>1</sup>	Set up a Function of two variables of the form $z = f(x, y)$ and then Do the Usual Routine		









# Outline

- Useful building blocks
  - ALU design
  - Register file
  - Memory considerations
- Datapath
- Design of the controller

# Warm-up

- A classic problem: sequence detection for “010” (non-overlapping)

Input:	0	1	0	0	1	0	1	0	1	1	0
Output:	0	0	0	1	0	0	1	0	0	0	0

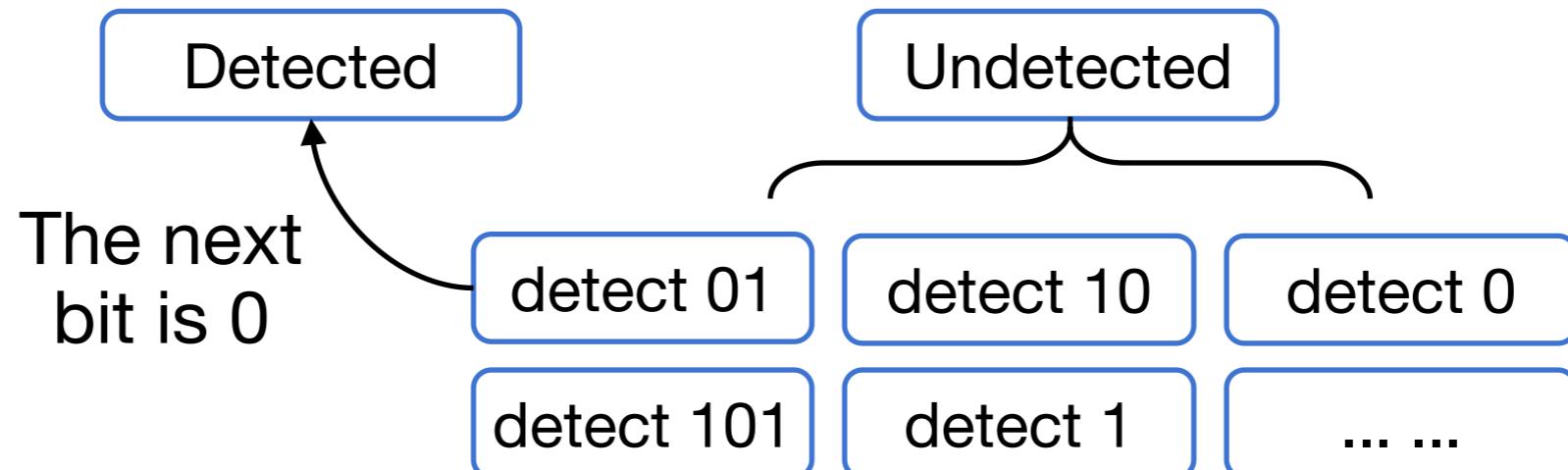
- Step 1: Draw finite state machine of the desired function (we ignore the initialization)
- Step 2: Define/assign binary numbers to represent the states, the inputs and the outputs
- Step 3: Write down the truth table (enumerate input/previous state (and current state) and their corresponding current state (and output))
- Step 4: Use template and decide the combinational block for state transition and output logic

# Warm-up

- A classic problem: sequence detection for “010” (non-overlapping)

Input: 0 1 0 0 1 0 1 0 1 1 0  
Output: 0 0 0 **1** 0 0 **1** 0 0 0 0

- Step 1: Draw finite state machine of the desired function

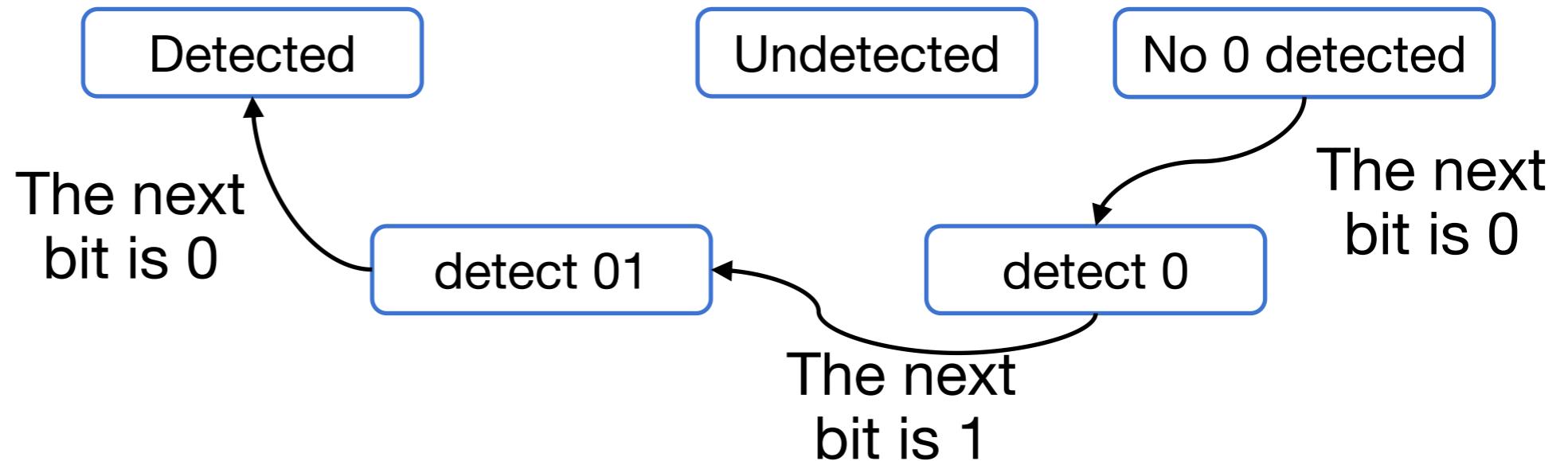


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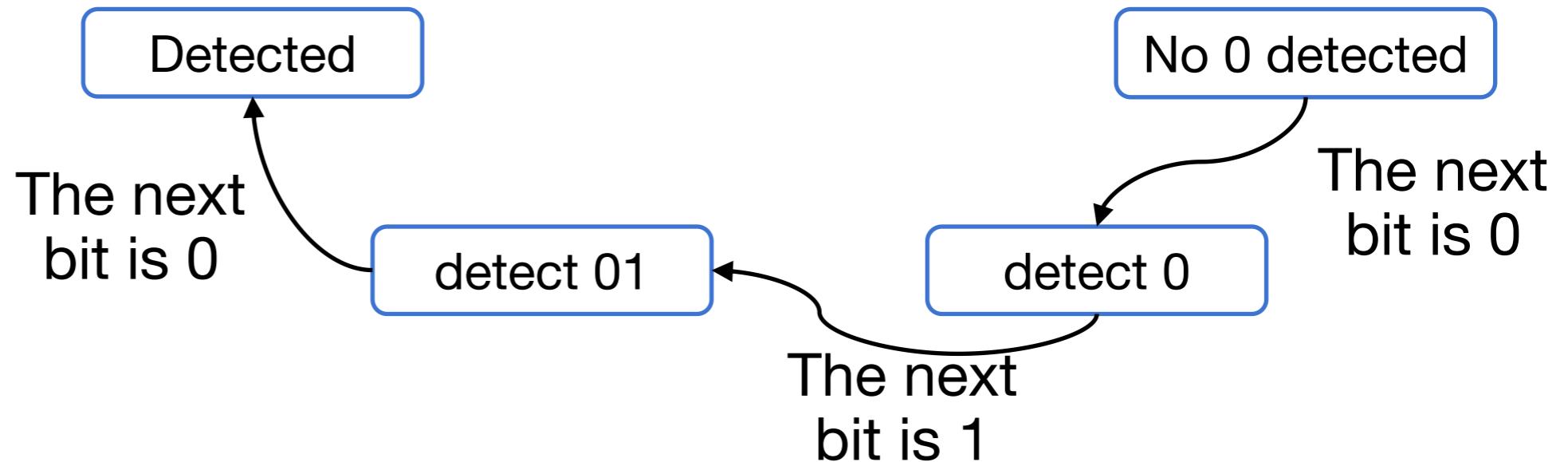


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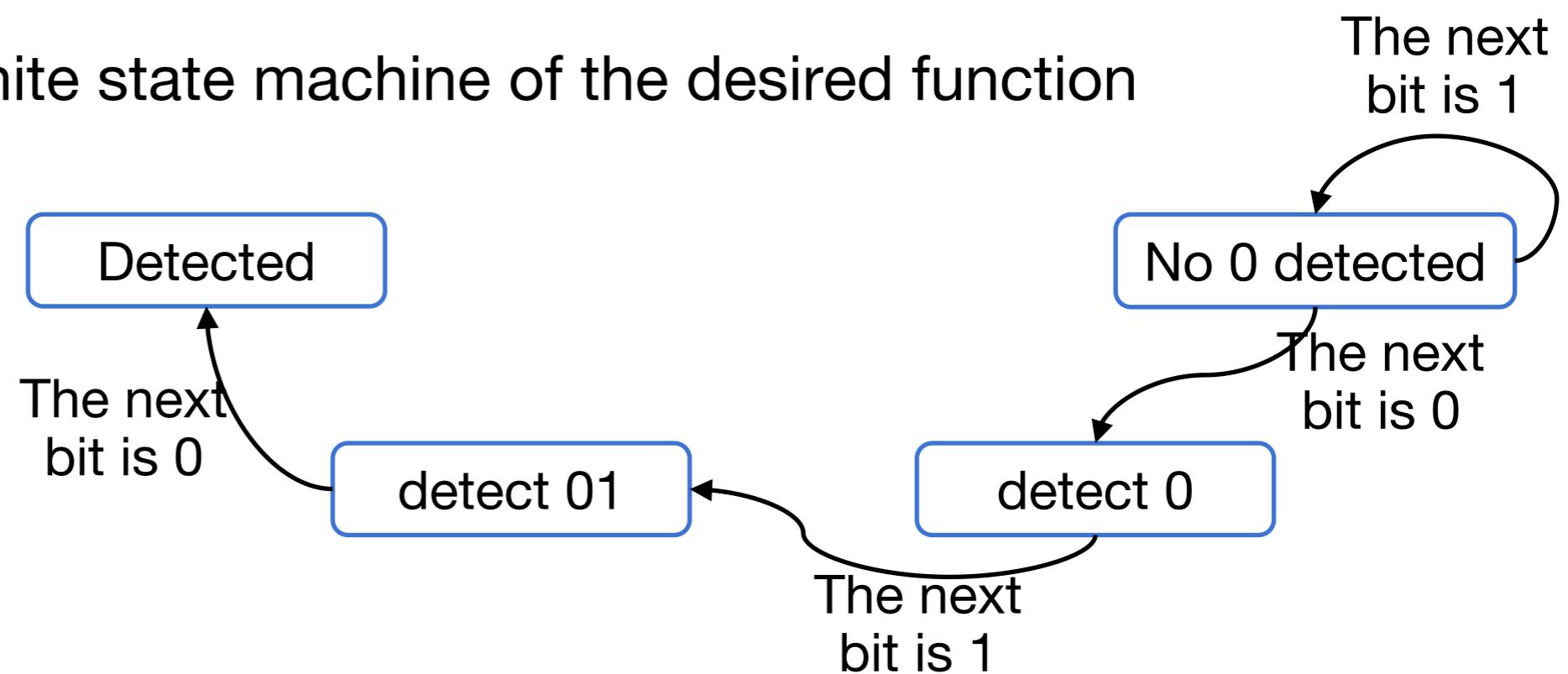


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Output: 0 0 0 **1** 0 0 **1** 0 0 0 0

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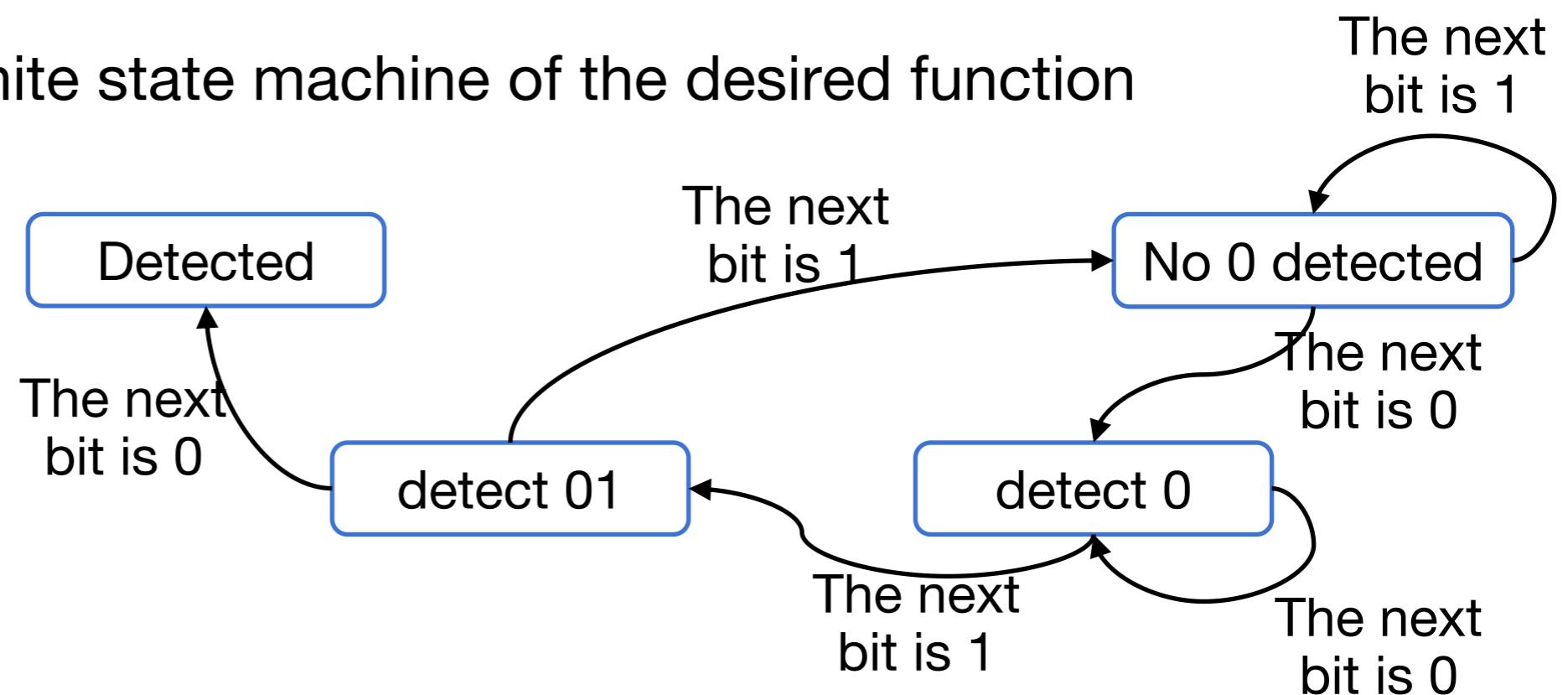


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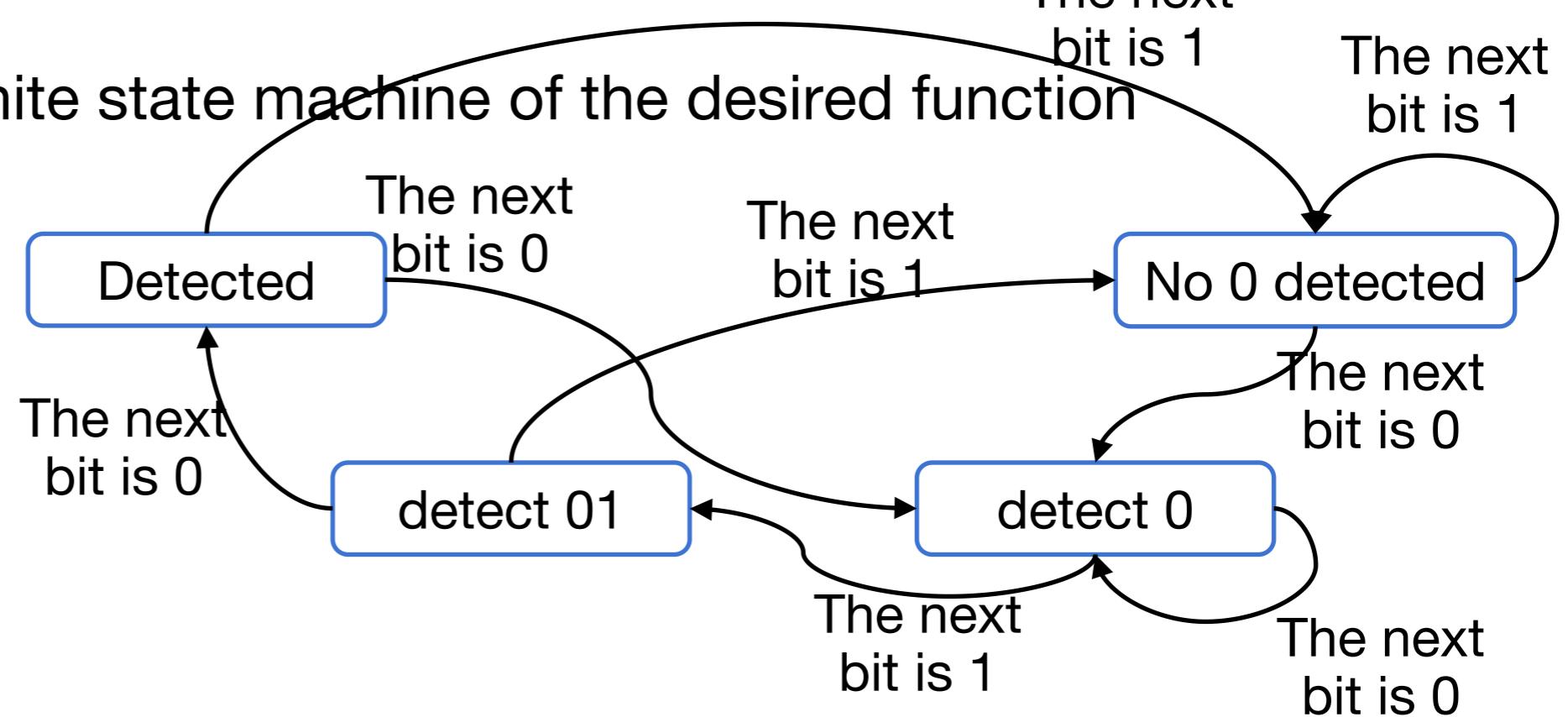
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Output: 0 0 0 **1** 0 0 **1** 0 0 0 0

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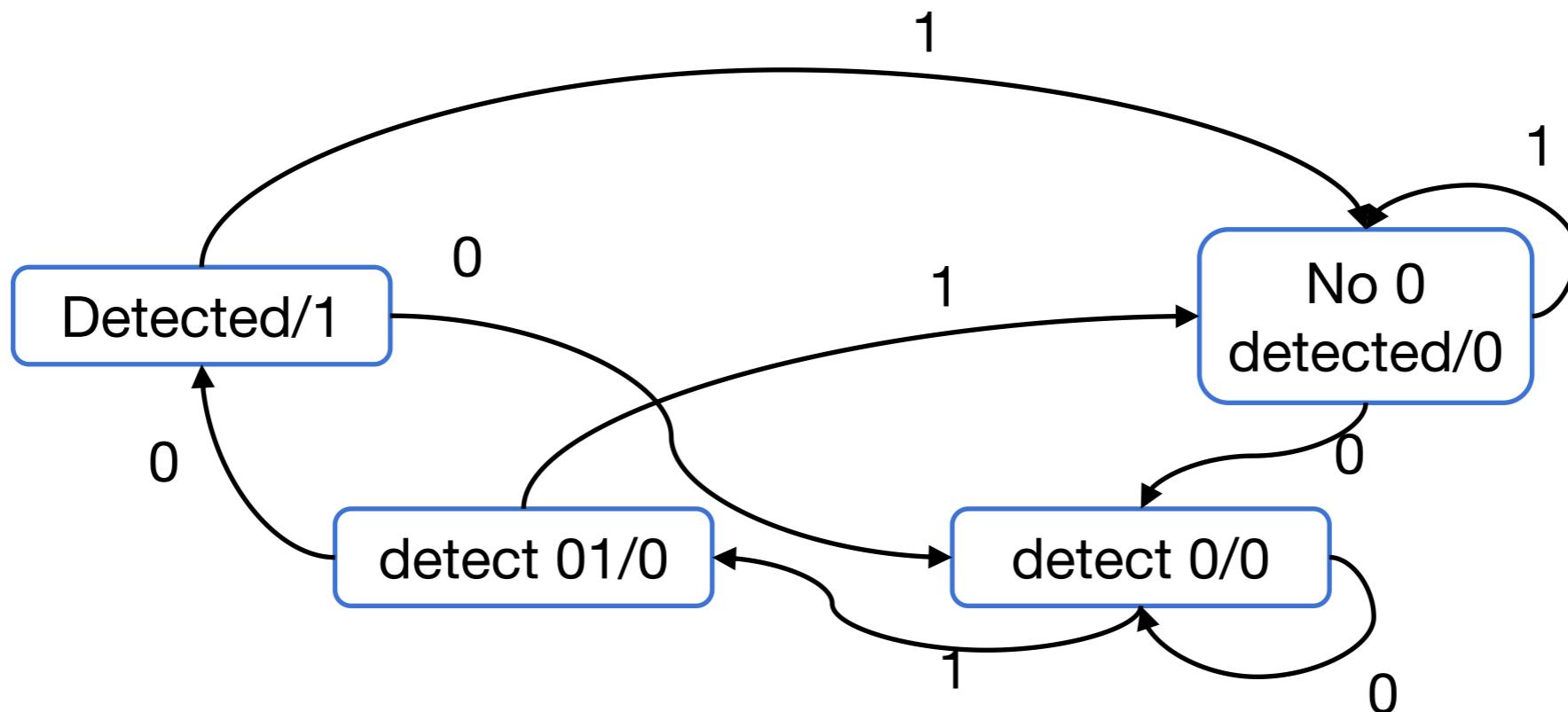


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- A classic problem: sequence detection for “010” (non-overlapping)

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Output: 0 0 0 **1** 0 0 **1** 0 0 0 0

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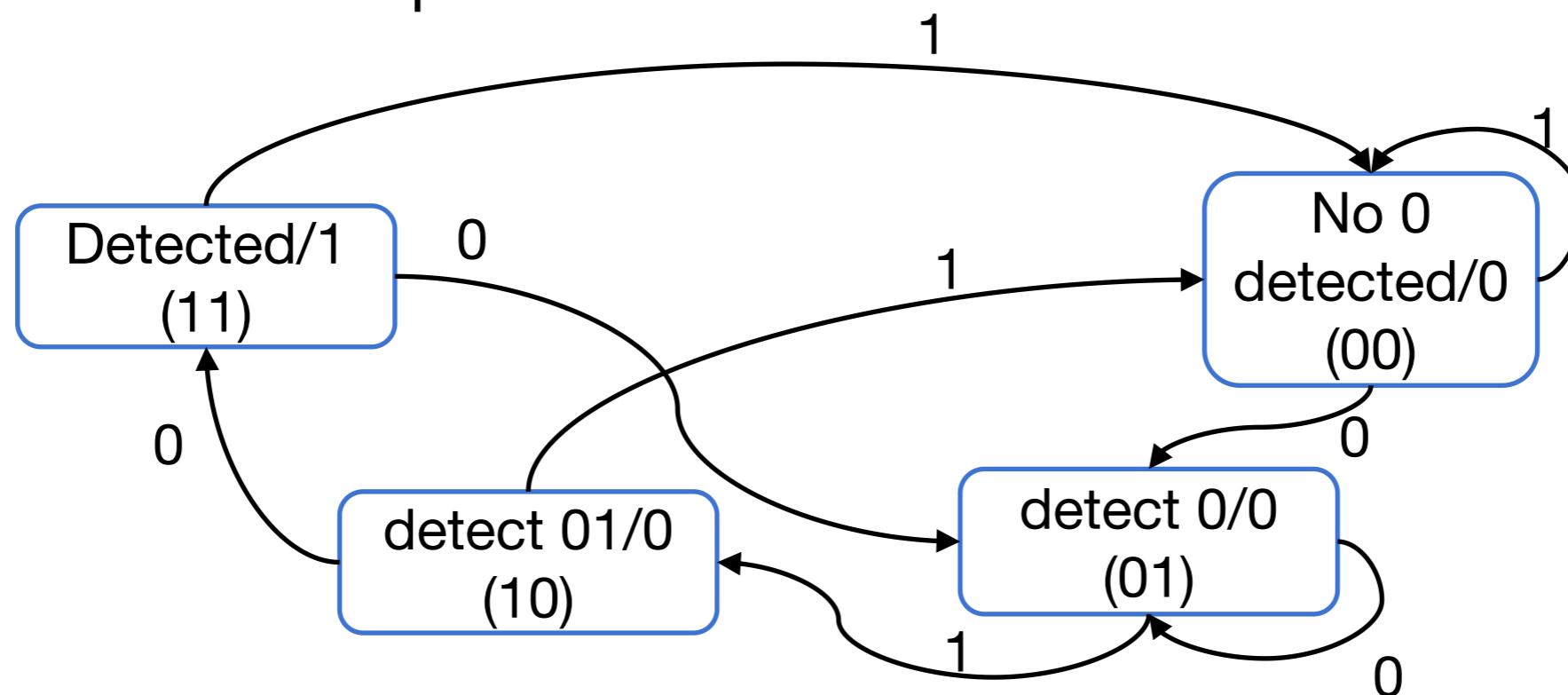


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Output: 0 0 0 **1** 0 0 **1** 0 0 0 0

- Step 2: Define/assign binary numbers to represent the states, the inputs and the outputs

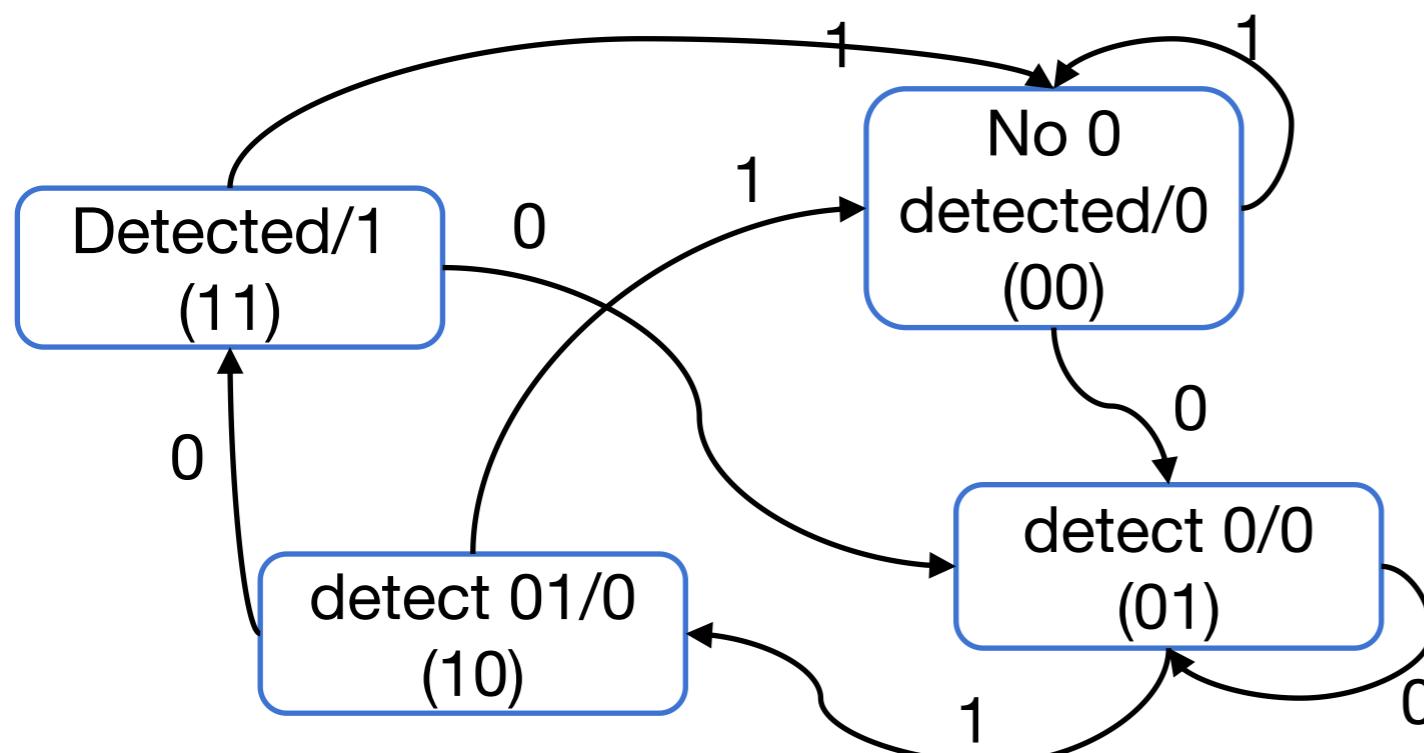


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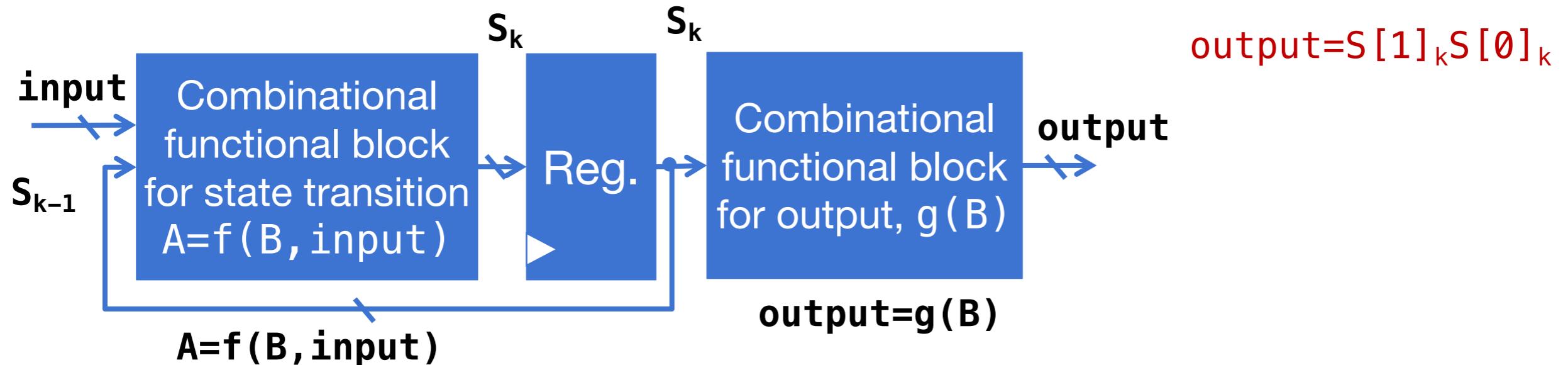
Input: 0 1 0 0 1 0 1 0 1 1 0  
 Output: 0 0 0 **1** 0 0 **1** 0 0 0 0

- Step 3: Write down the truth table (enumerate input/previous state (and current state) and their corresponding current state (and output))

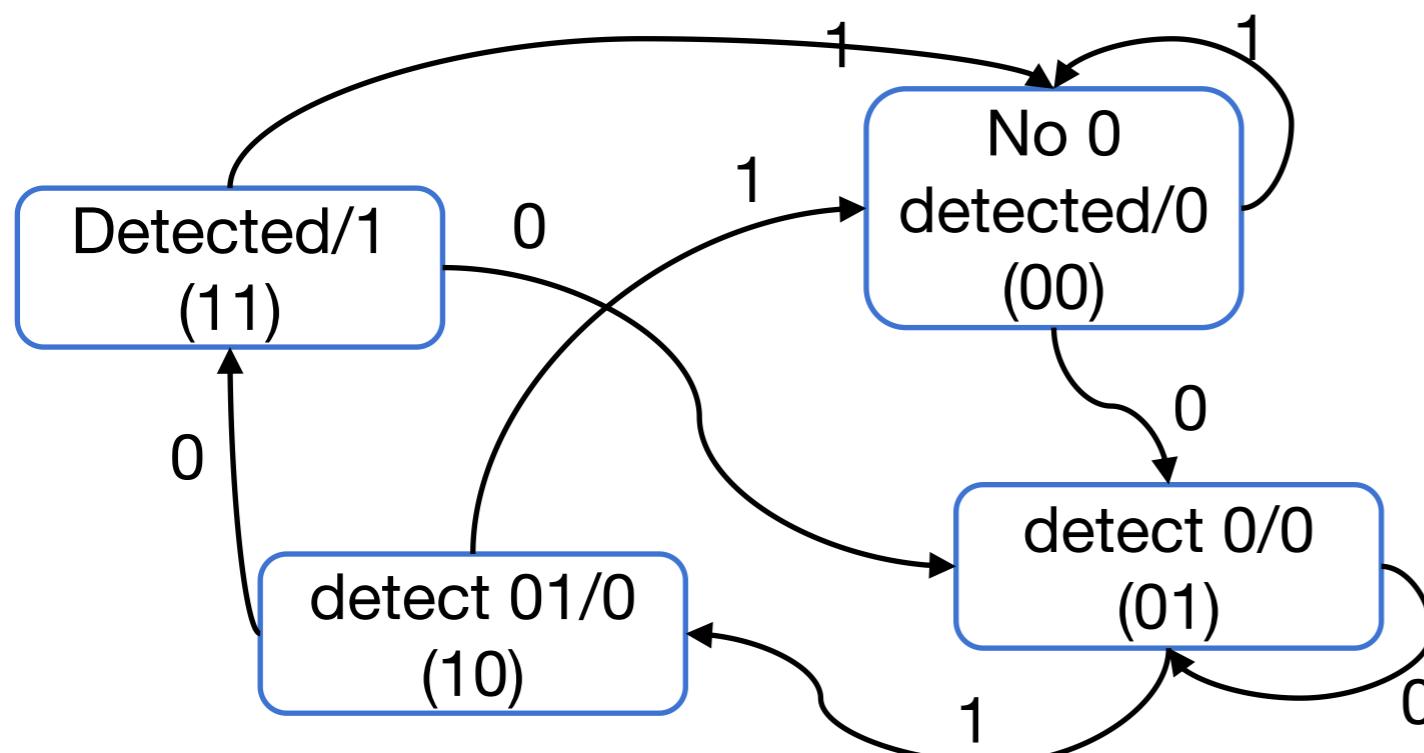


input	Previous state S[1] k-1	Previous state S[0] k-1	Current state S[1] k	Current state S[0] k	output
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	0

# Warm-up

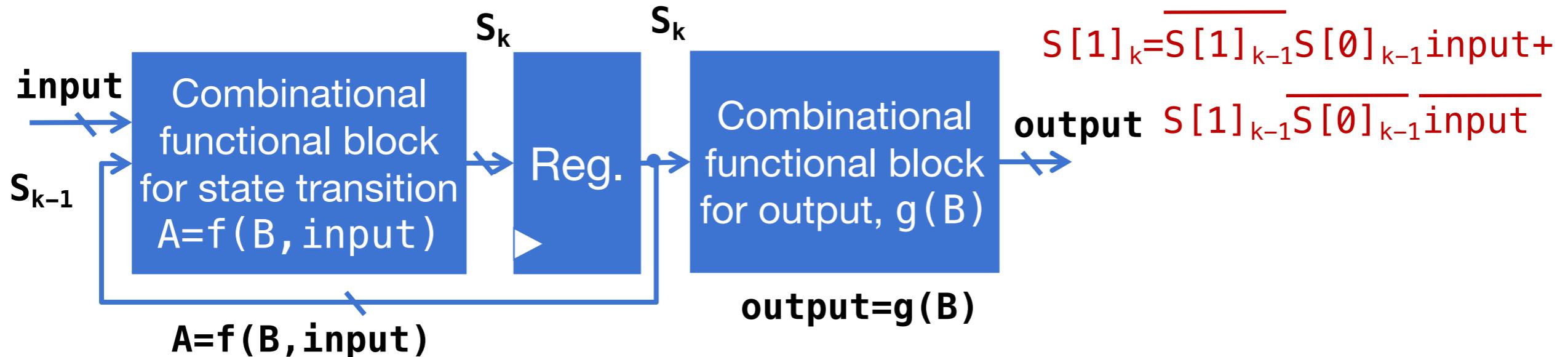


- Step 4: Use template and decide the combinational block for state transition and output logic

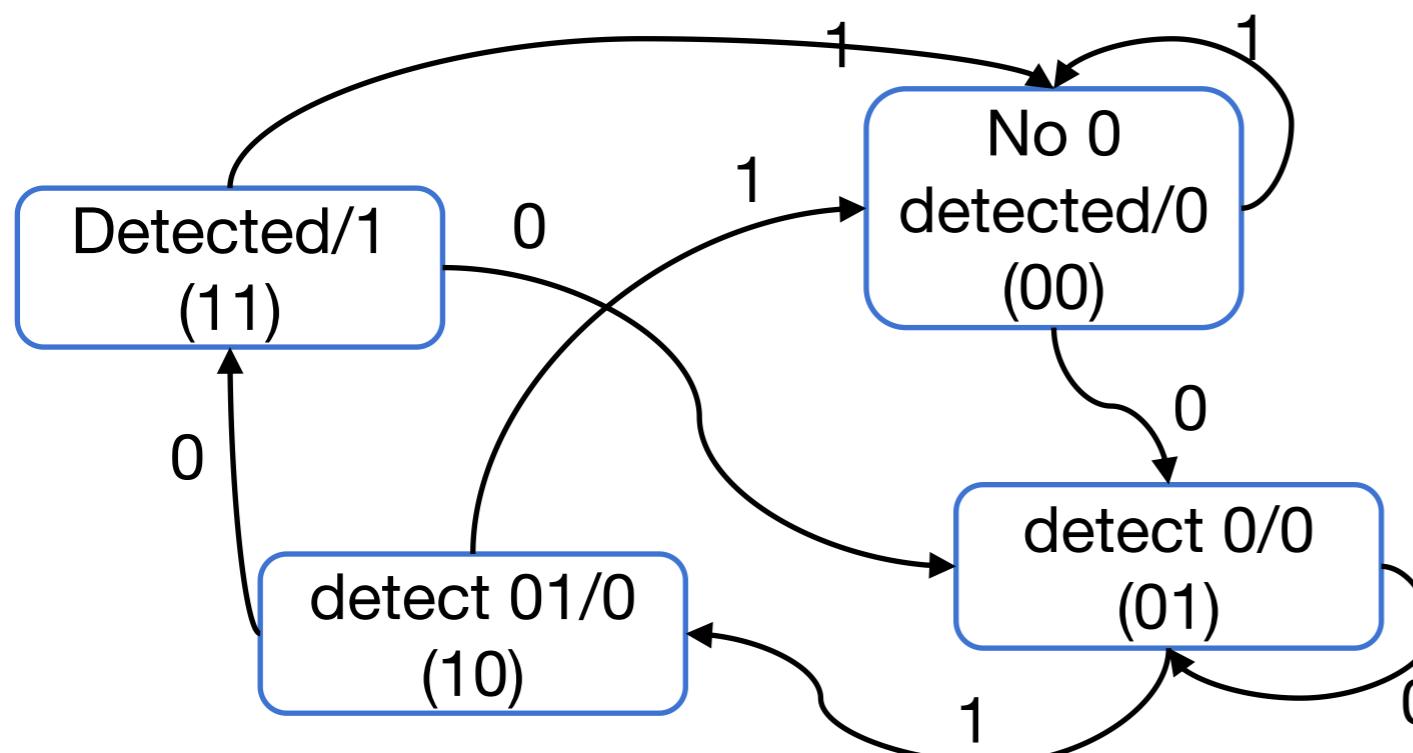


input	Previous state		Current state		output
	$S[1]_{k-1}$	$S[0]_{k-1}$	$S[1]_k$	$S[0]_k$	
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	0

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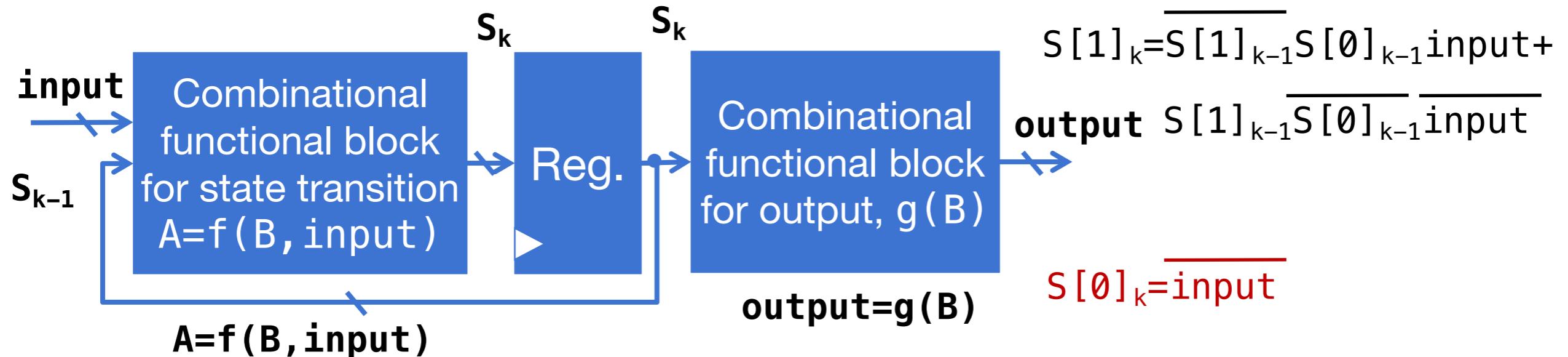


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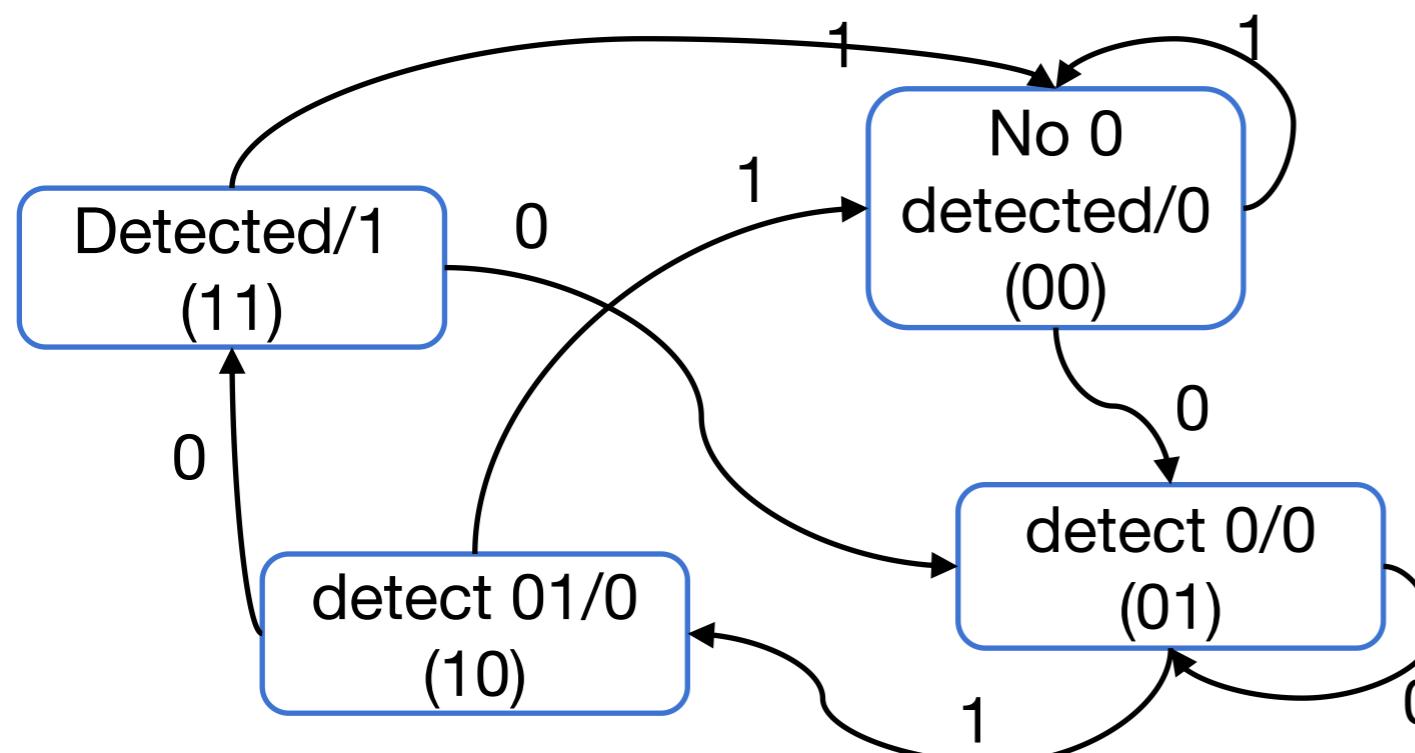


	Previous state	Current state			
input	$S[1]_{k-1}$	$S[0]_{k-1}$	$S[1]_k$	$S[0]_k$	output
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	0

# Warm-up

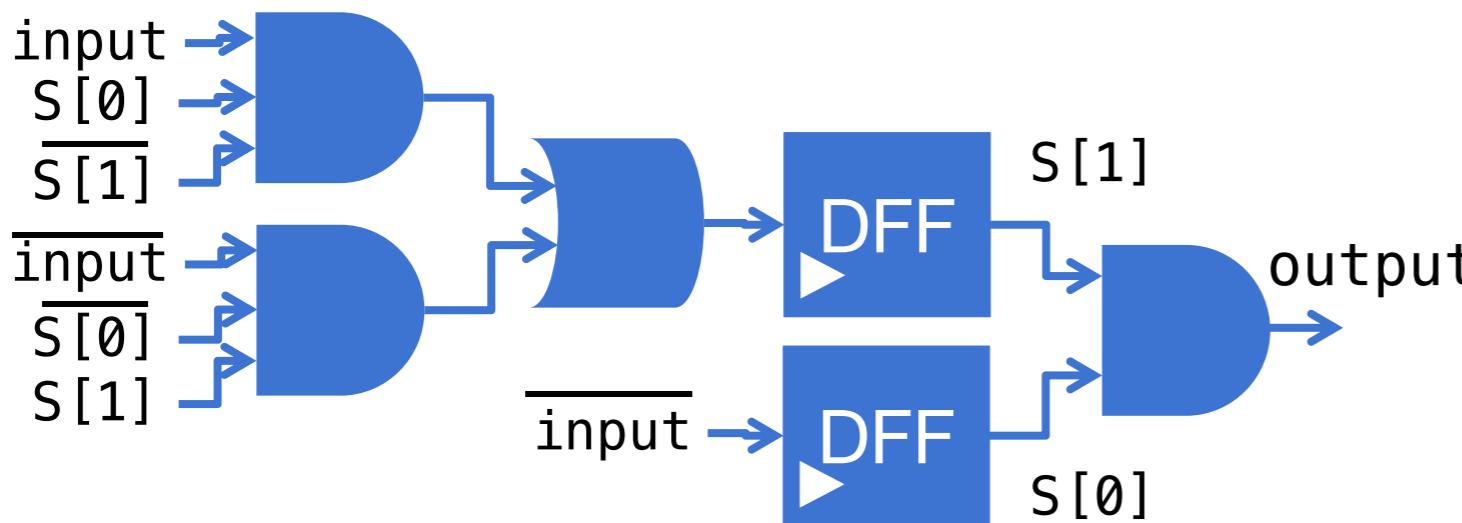


- Step 4: Use template and decide the combinational block for state transition and output logic



	Previous state	Current state			
input	$S[1]_{k-1}$	$S[0]_{k-1}$	$S[1]_k$	$S[0]_k$	output
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	0

# Warm-up



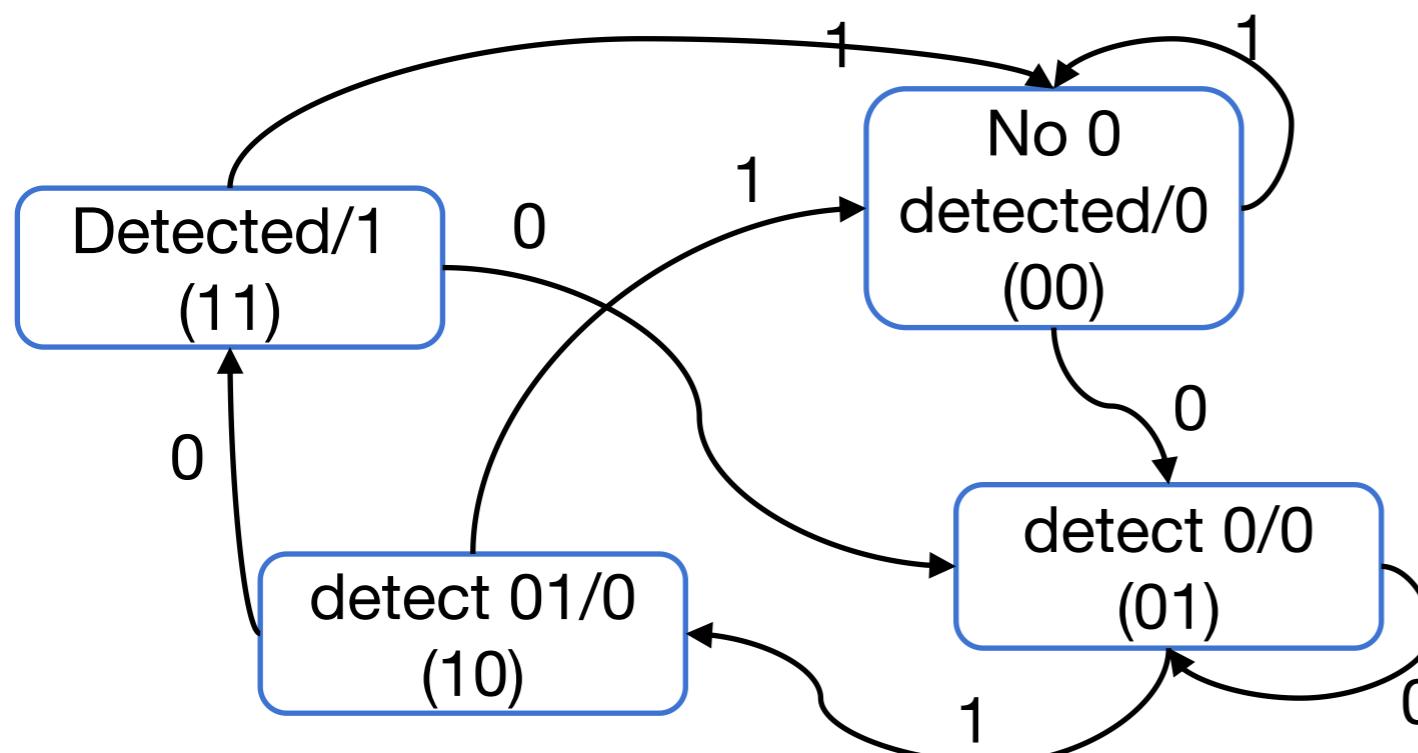
$$\text{output} = S[1]_k S[0]_k$$

$$S[1]_k = \overline{S[1]_{k-1} S[0]_{k-1} \text{input}} +$$

$$S[1]_{k-1} \overline{S[0]_{k-1} \text{input}}$$

$$S[0]_k = \overline{\text{input}}$$

- Step 4: Use template and decide the combinational block for state transition and output logic

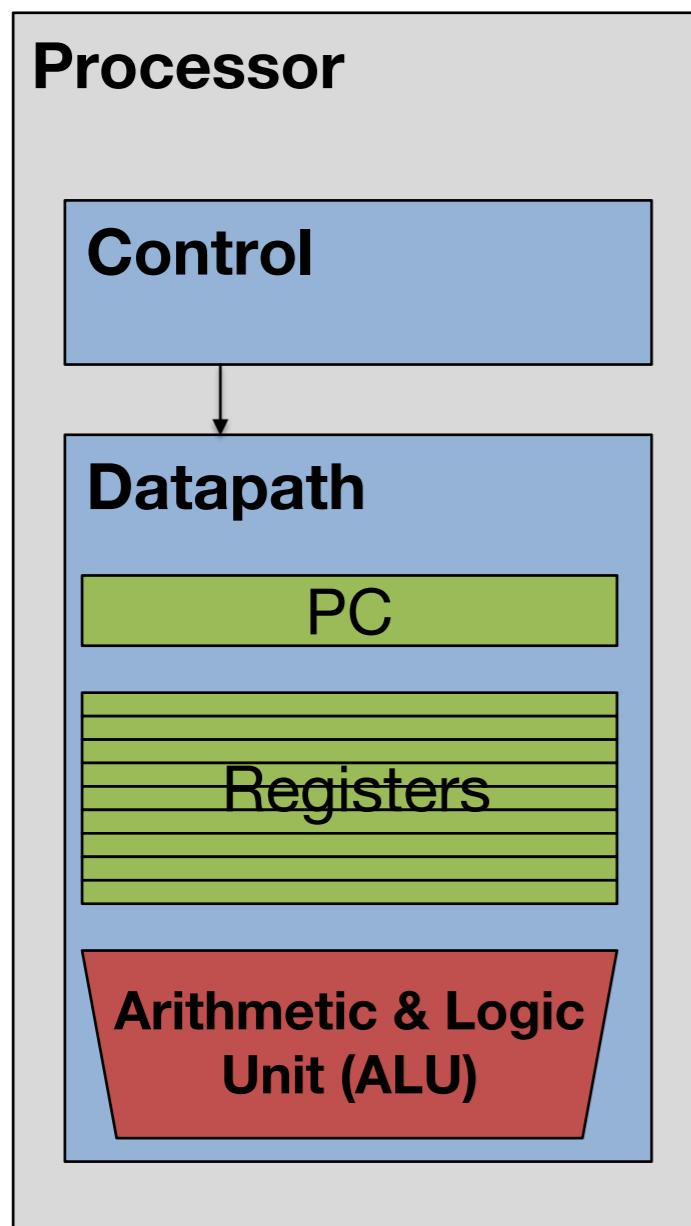


Previous state      Current state

input	$S[1]_{k-1}$	$S[0]_{k-1}$	$S[1]_k$	$S[0]_k$	output
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	0

# Controller & Datapath

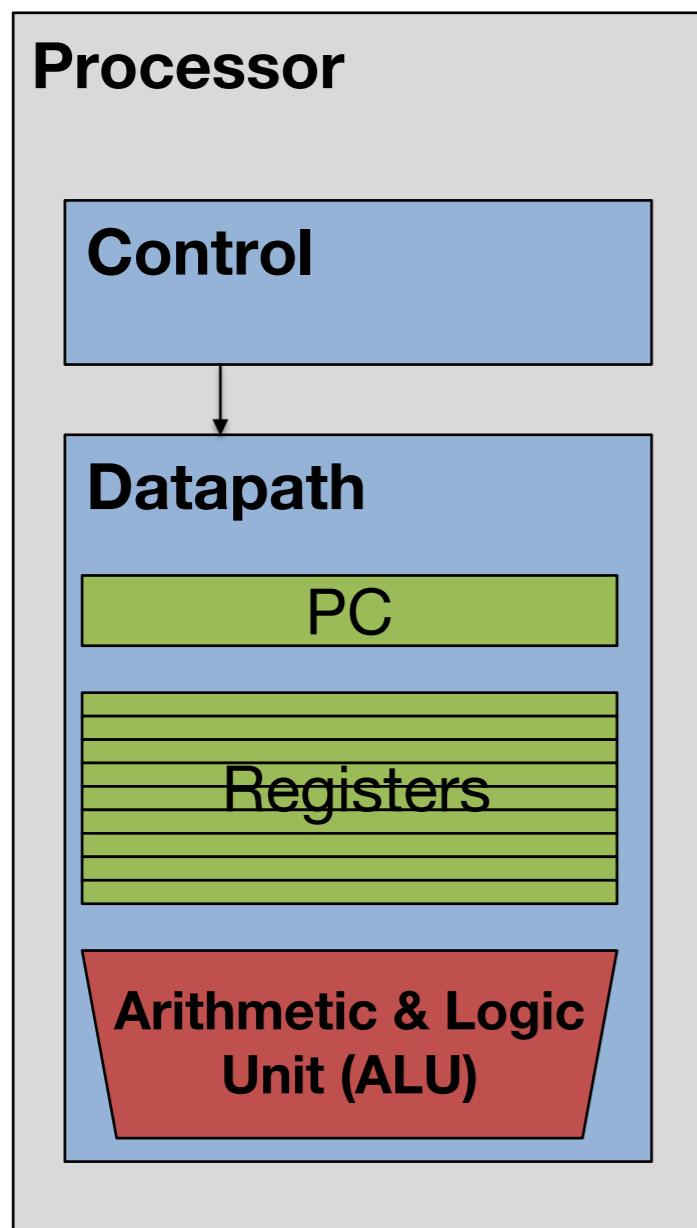
- A CPU that support RV32I can have so many states



- Consider the 32 registers alone
  - $x_0$  always 0
  - Each bit in the other registers can be 0 or 1
- Not practical to enumerate all the state transitions
- Top-down design: build small modules and then connect them as needed
- Most digital systems can be divided into datapath and controller
  - Datapath contains data processing and storage
  - Controller controls data access (still can be modeled as FSM)
- Recall the execution of an instruction
  - Our Goal: Implement a RISC-V processor as a synchronous digital system (SDS).
  - Each RV32I instruction can be done within 1 clock cycle (single-cycle CPU).

# Controller & Datapath

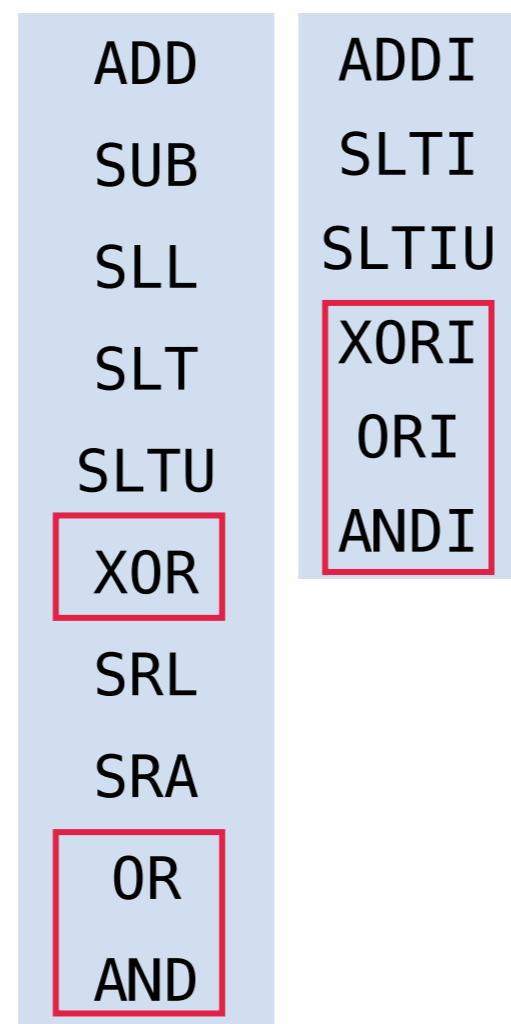
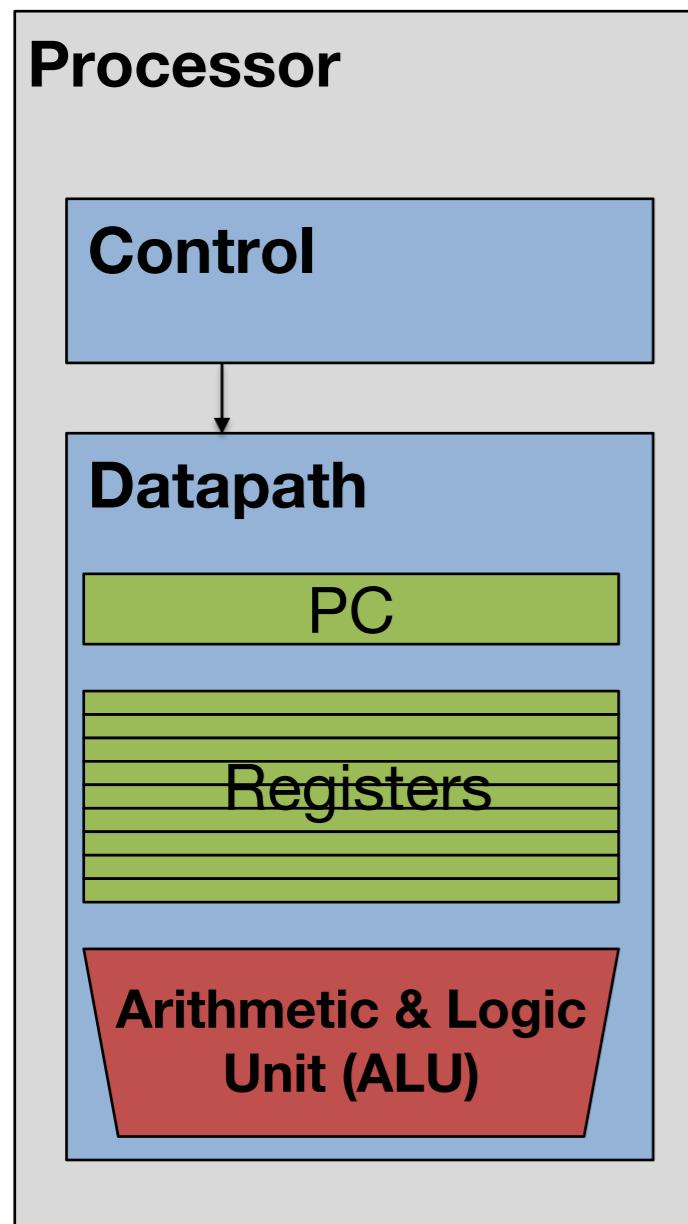
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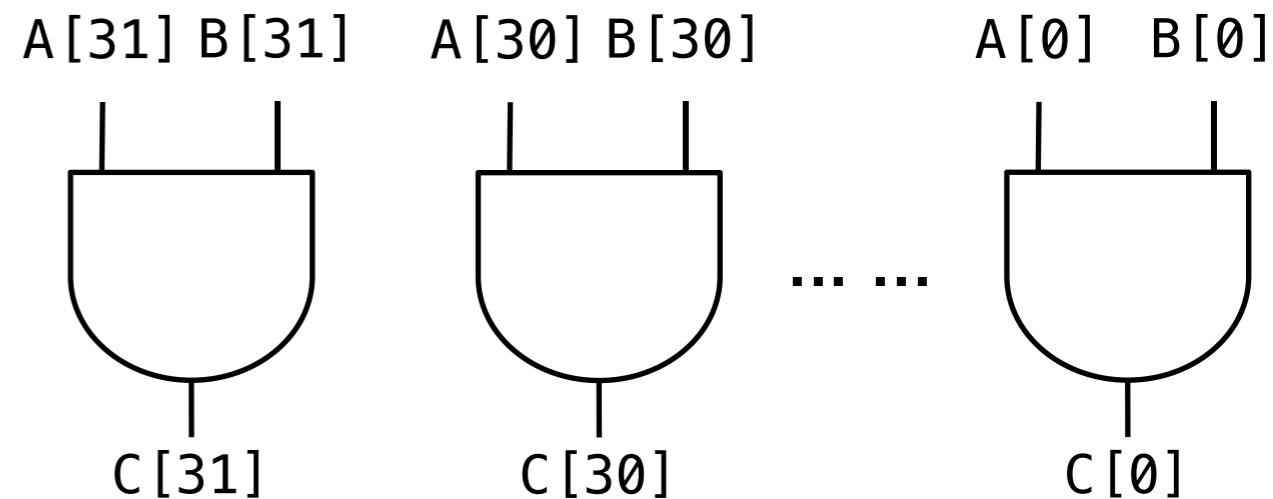
- Datapath
  - Start with basic building blocks
  - Add building blocks to the digital system with added supported instructions
- Controller
  - Can be considered as an FSM
- Our Goal: Implement a RISC-V processor as a synchronous digital system (SDS).
- Each RV32I instruction can be done within 1 clock cycle (single-cycle CPU).

# Useful building blocks

- An ALU should be able to execute all the arithmetic and logic operations



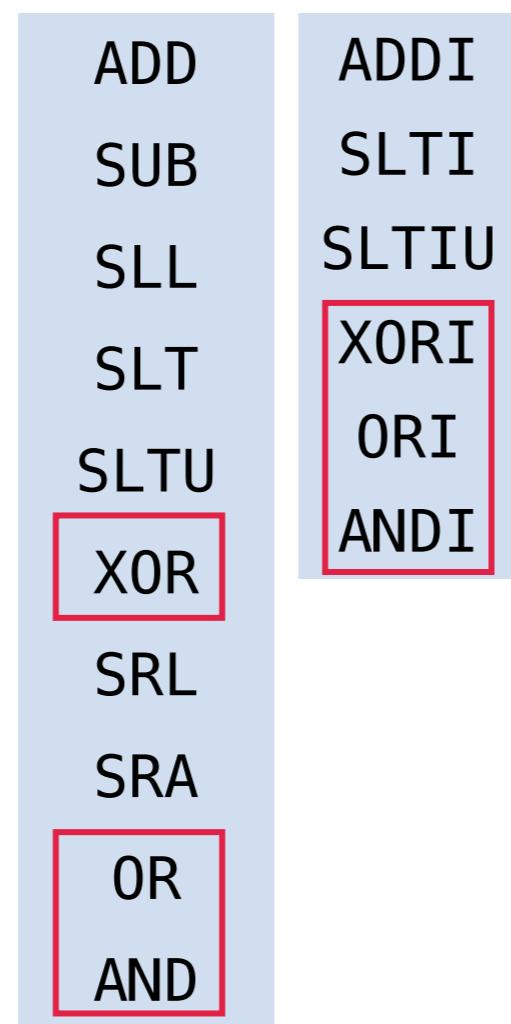
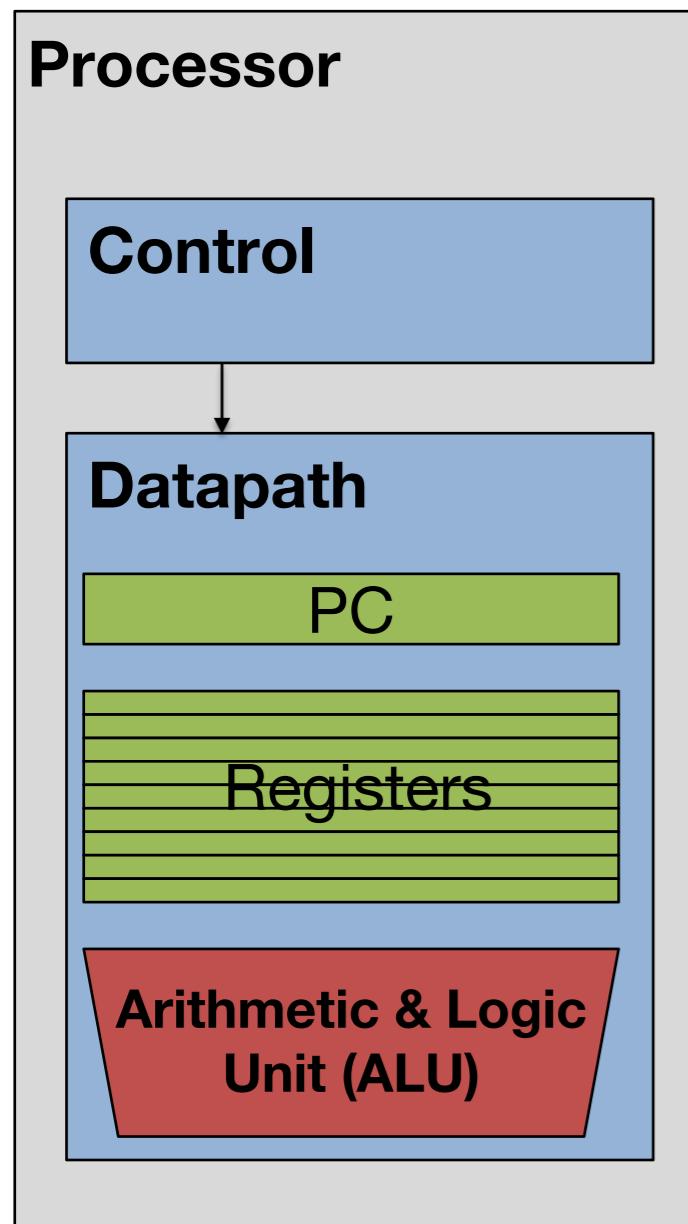
- AND as an example
  - 2 32-bit inputs A and B
  - 1 32-bit output C



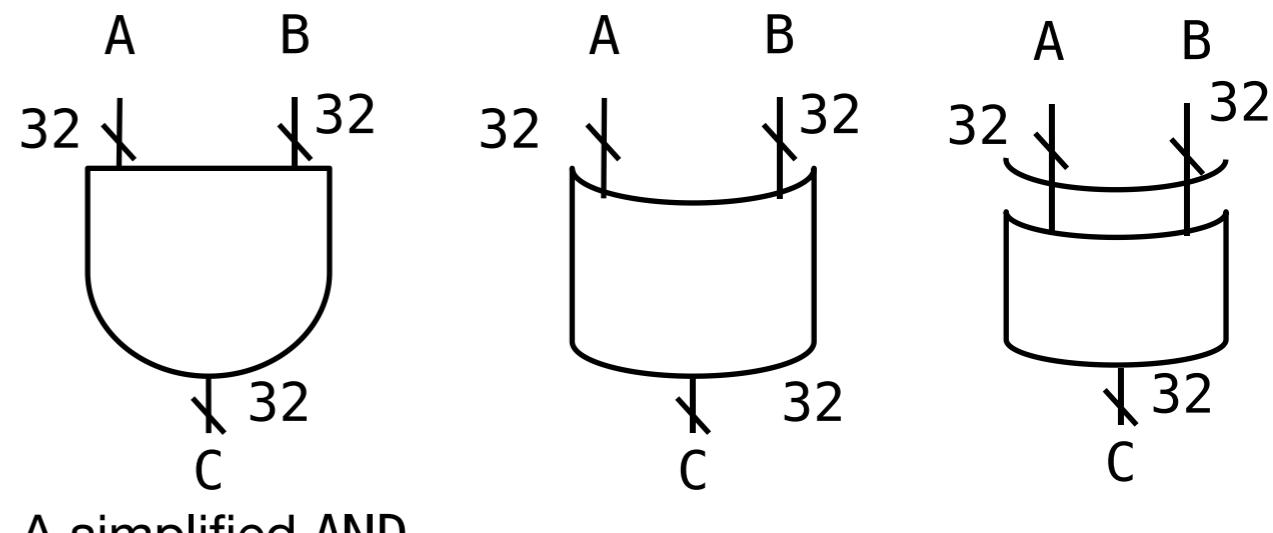
- Our Goal: Implement a RISC-V processor as a synchronous digital system.
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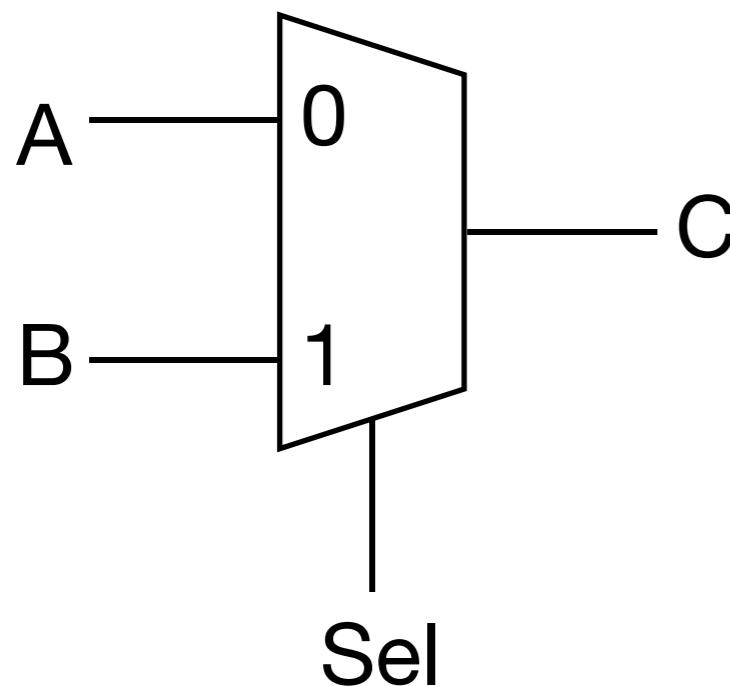


A simplified AND gate array symbol

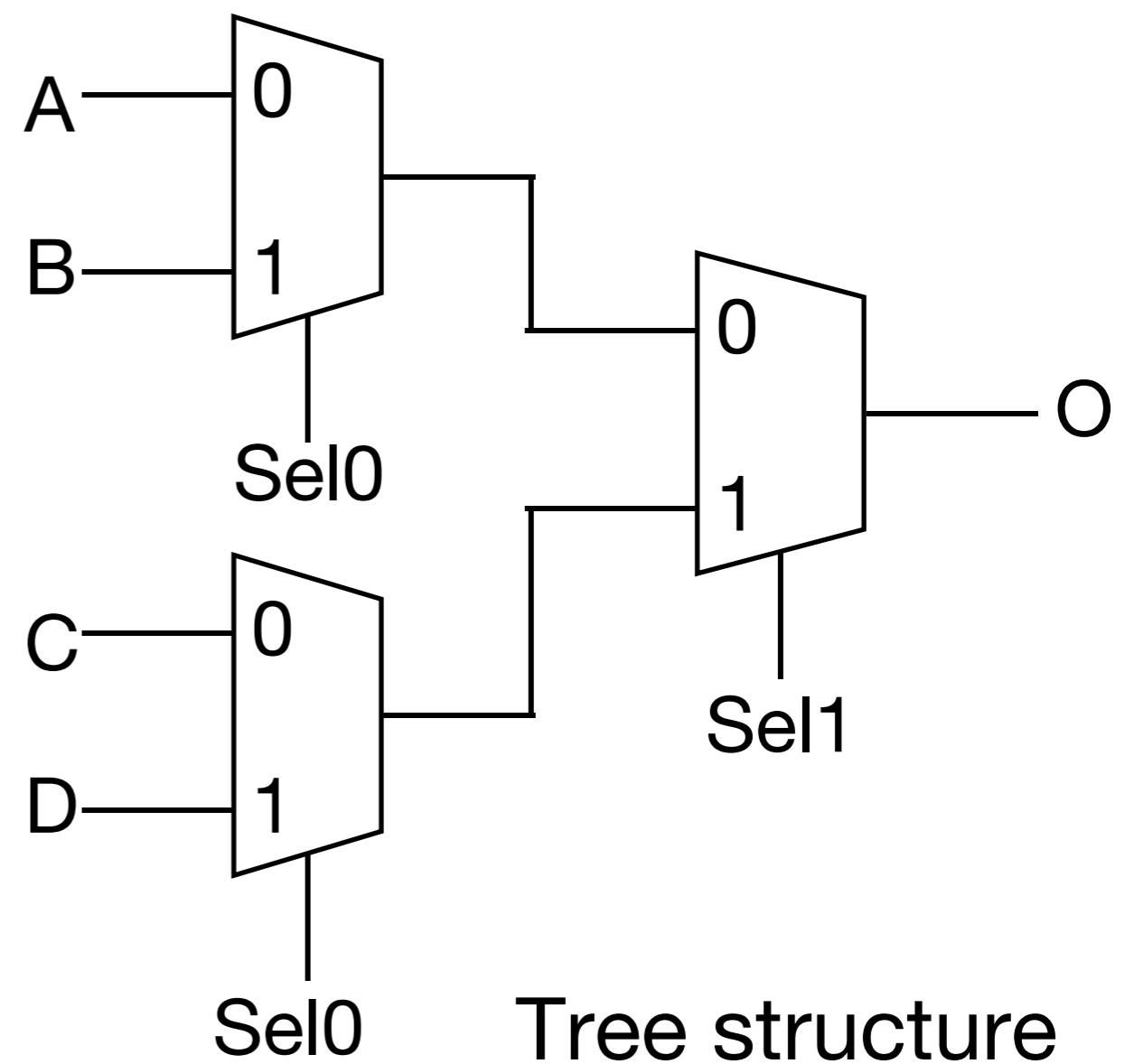
- Our Goal: Implement a RISC-V processor as a synchronous digital system.
- Each RV32I instruction can be done within 1 clock cycle.

# Useful Combinational Circuits

- Multiplexer (2-to-1)

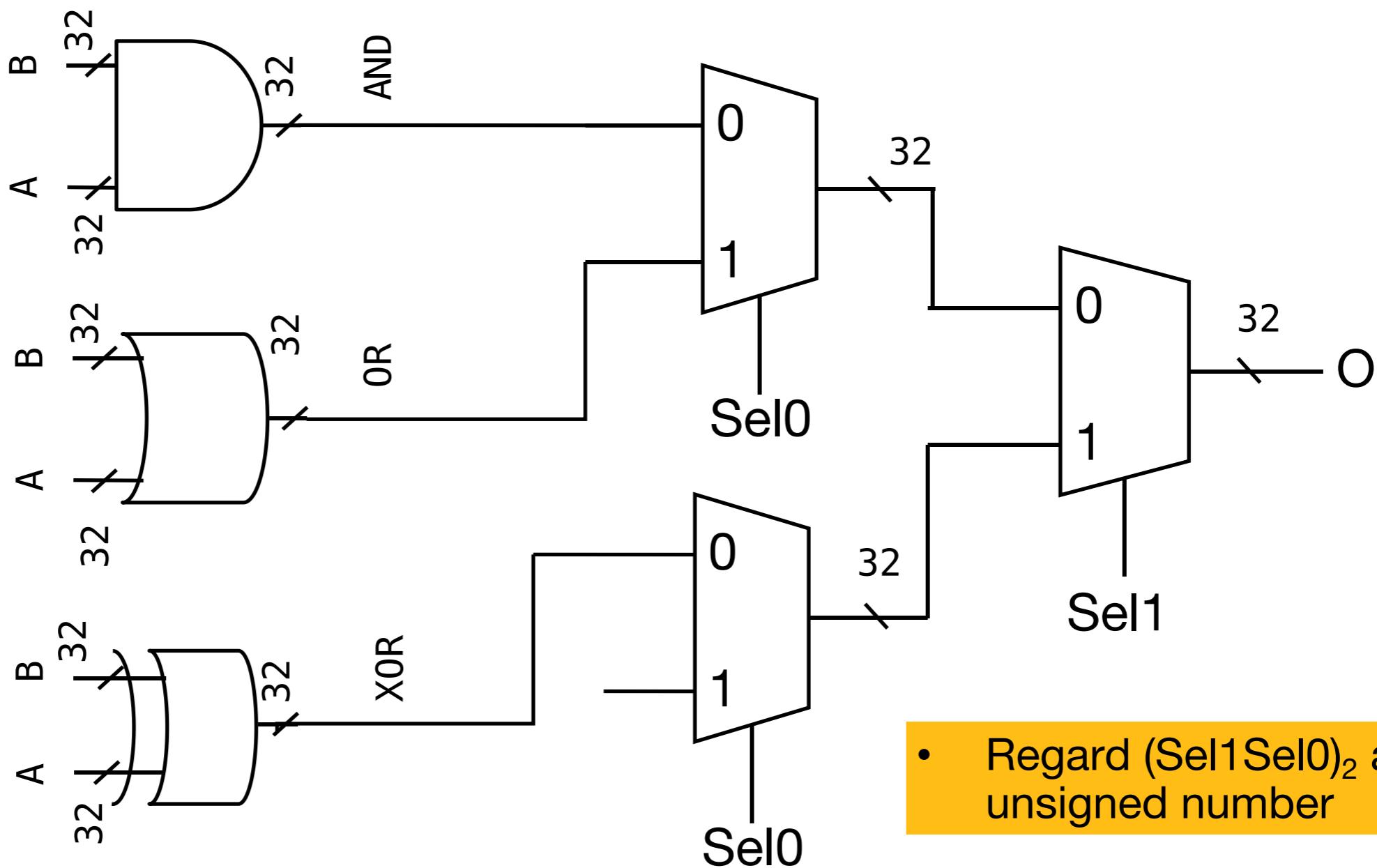


- Multiplexer ( $2^n$ -to-1)



# Control through selection

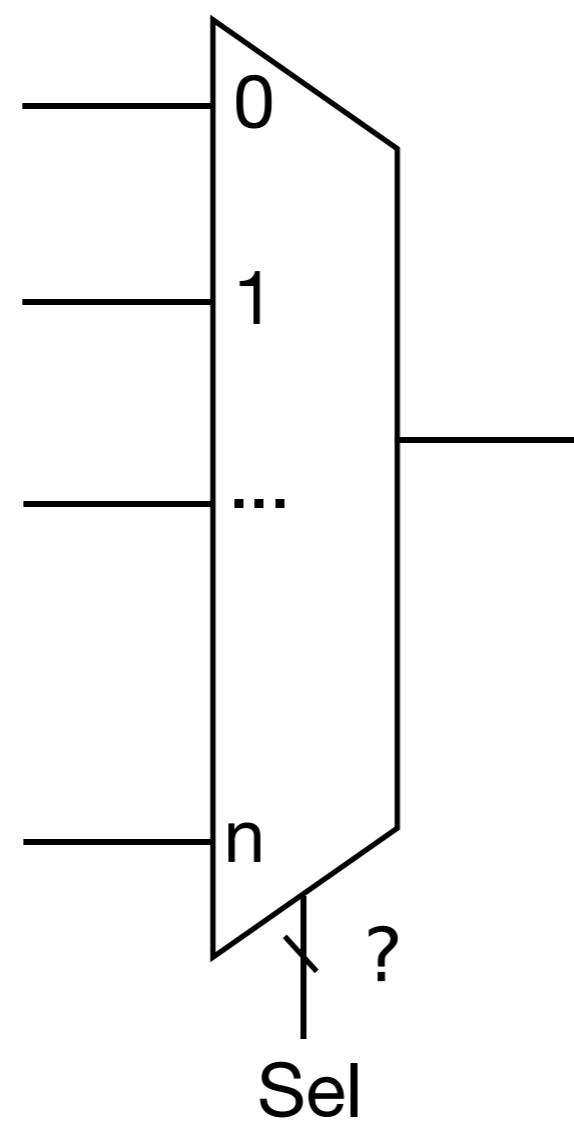
- 32-bit Multiplexer and logic gates to support some logic instructions



- More layers of multiplexer to select from more inputs

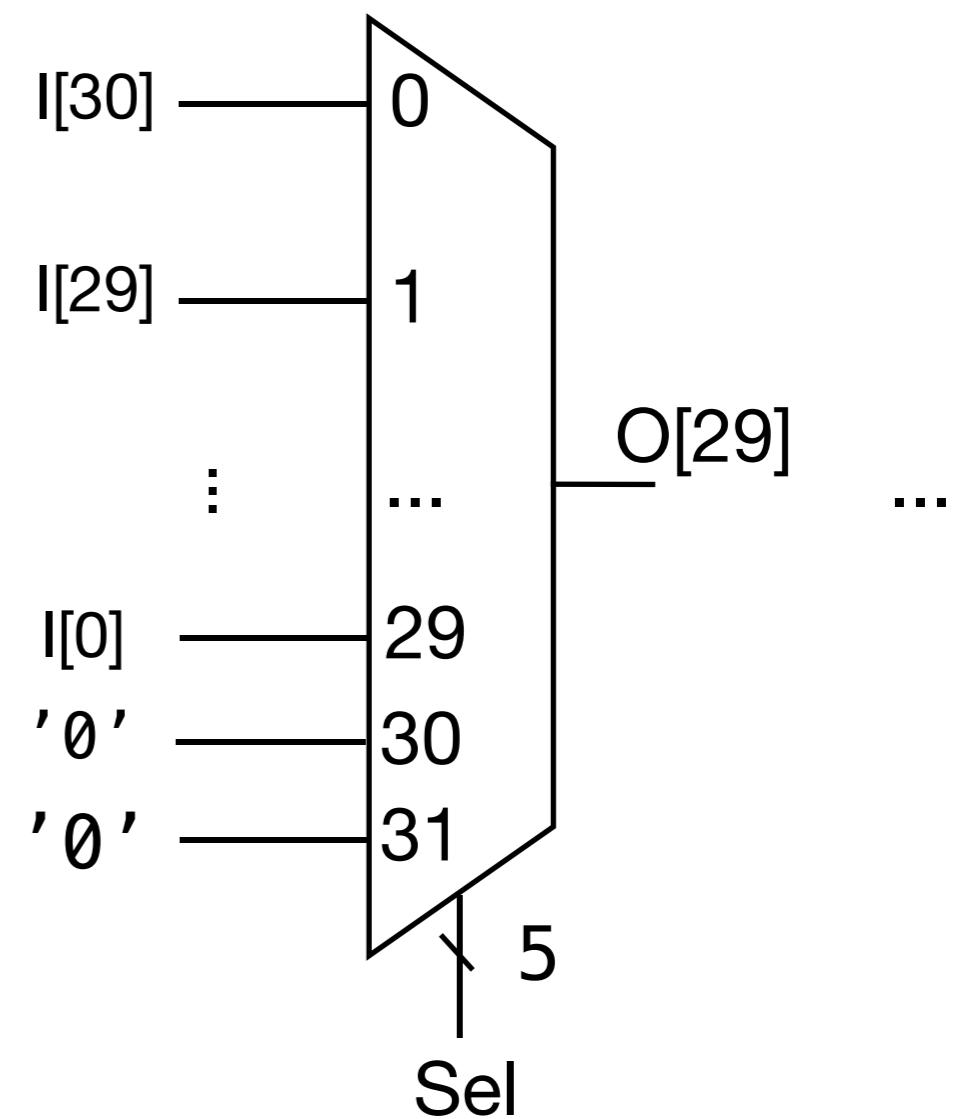
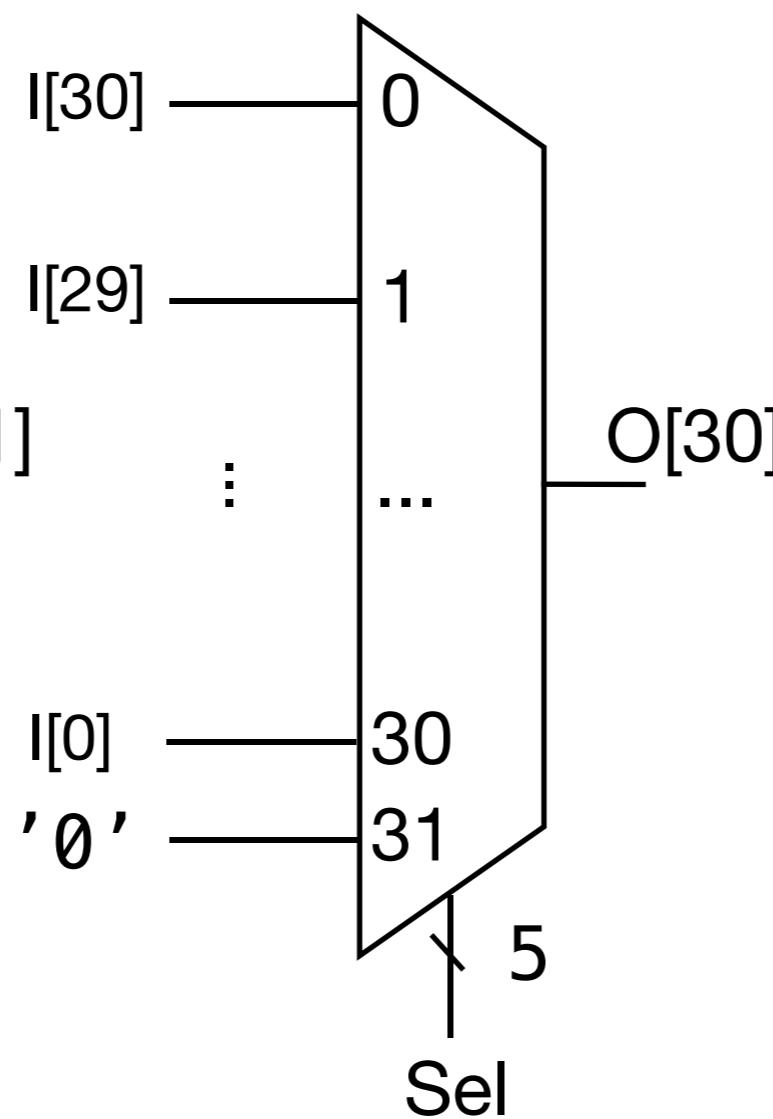
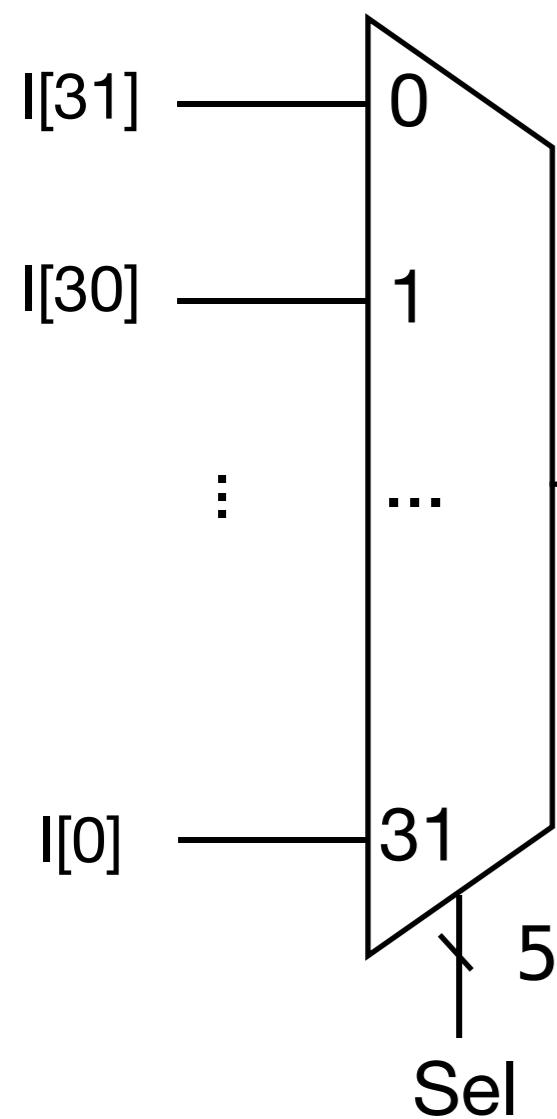
# Multiplexer

- N-to-1 multiplexer symbol



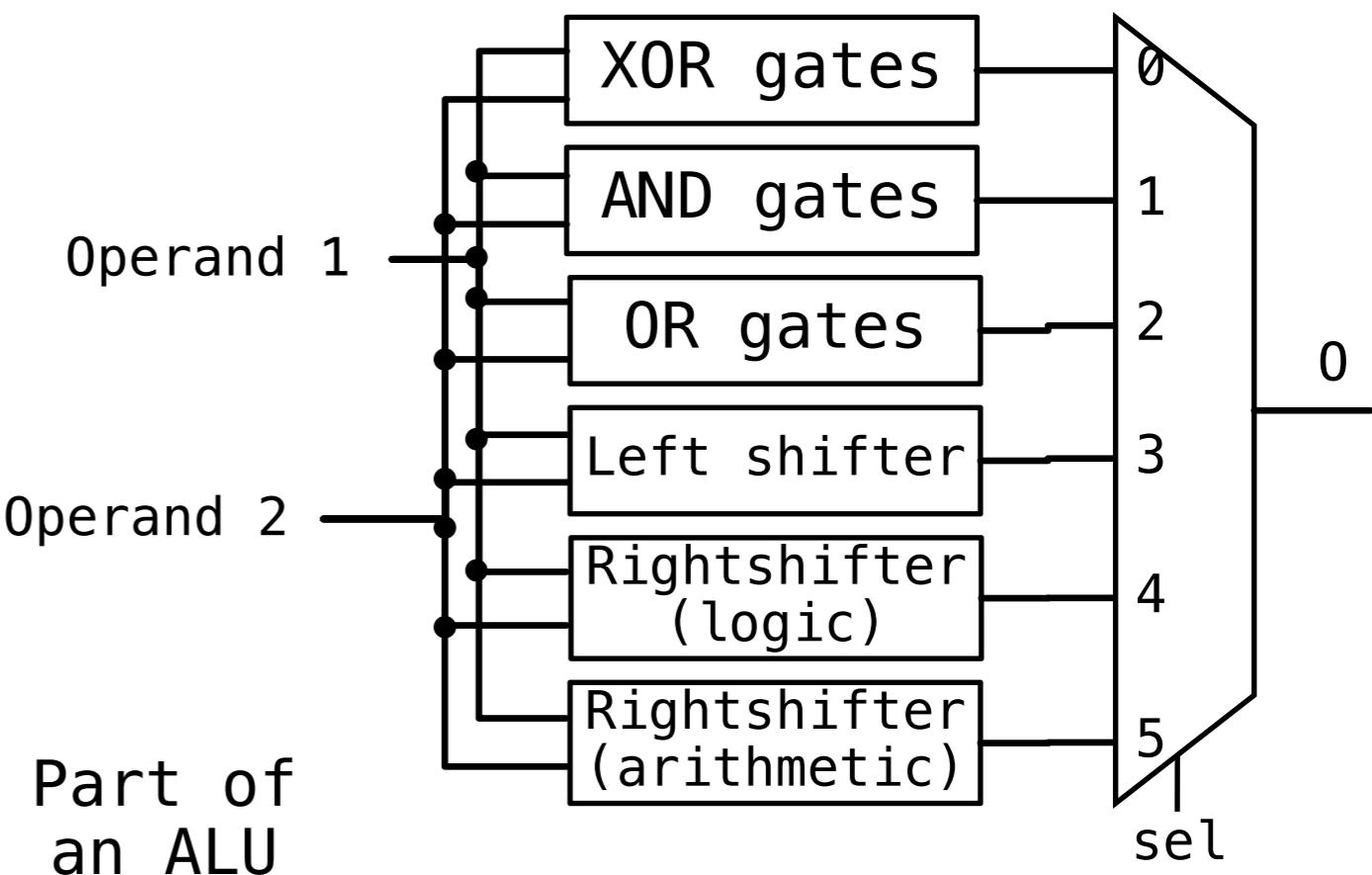
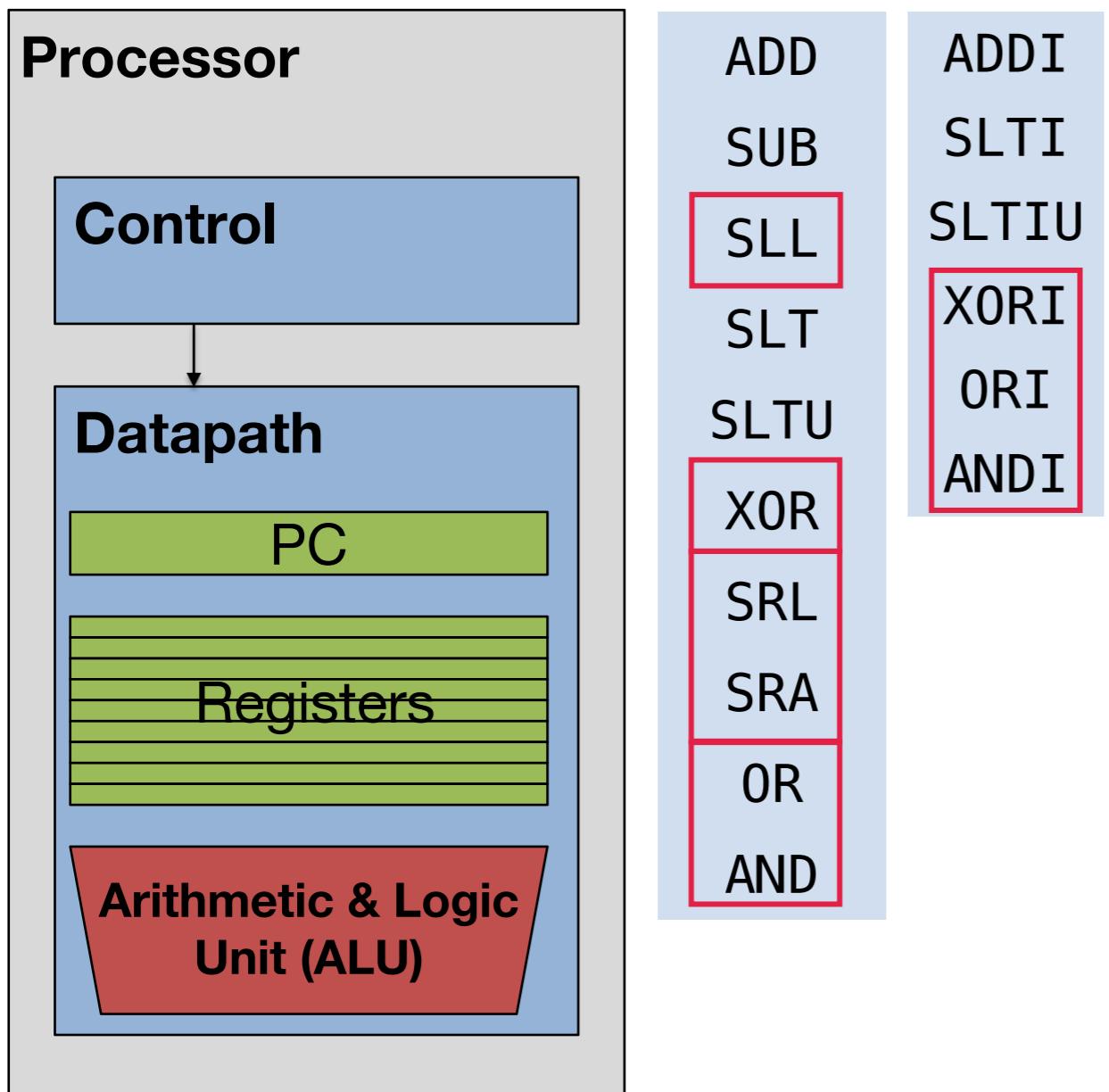
# Multiplexers used for shifter

- Left shift a single bit -> left shift multiple single bits
- Other shifter designs such as barrel shifter



# Useful building blocks

- An ALU should be able to execute all the arithmetic and logic operations

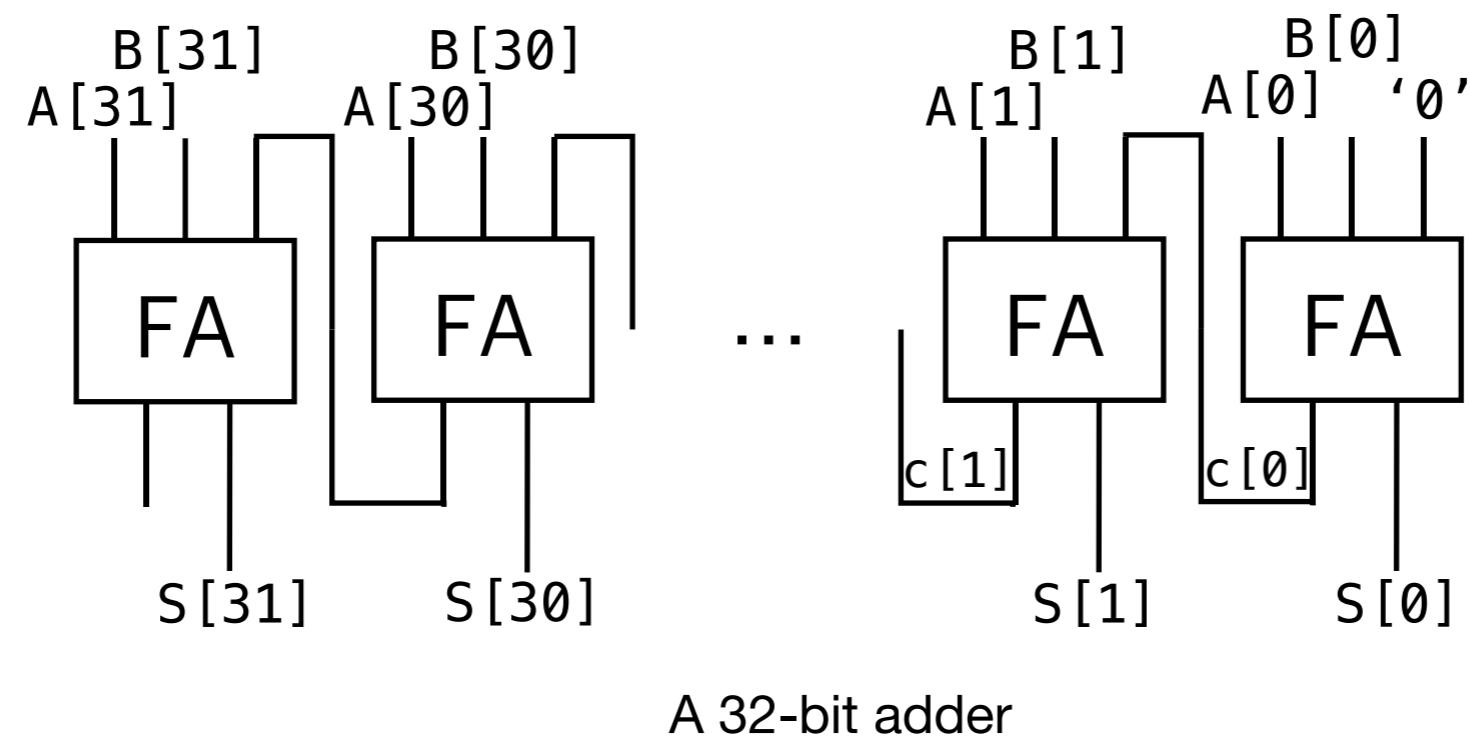


Note that all the signals expect the selection signals are 32-bit.

- Our Goal: Implement a RISC-V processor as a synchronous digital system.
- Each RV32I instruction can be done within 1 clock cycle.

# Adder & subtractor

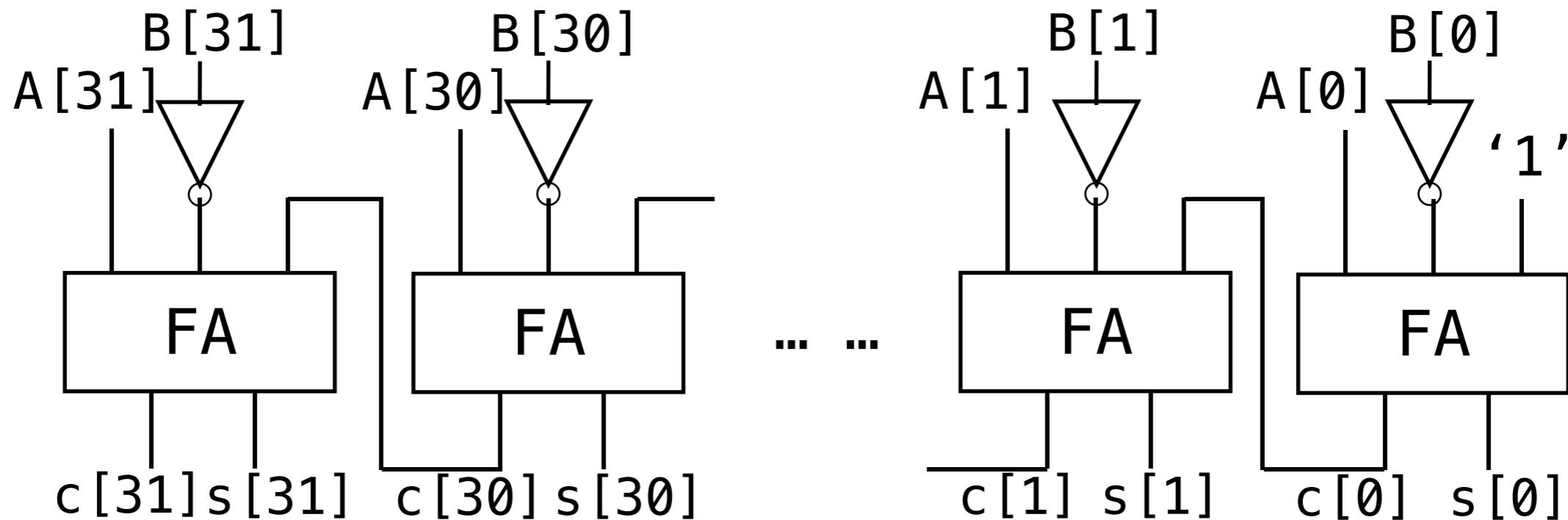
- An adder design



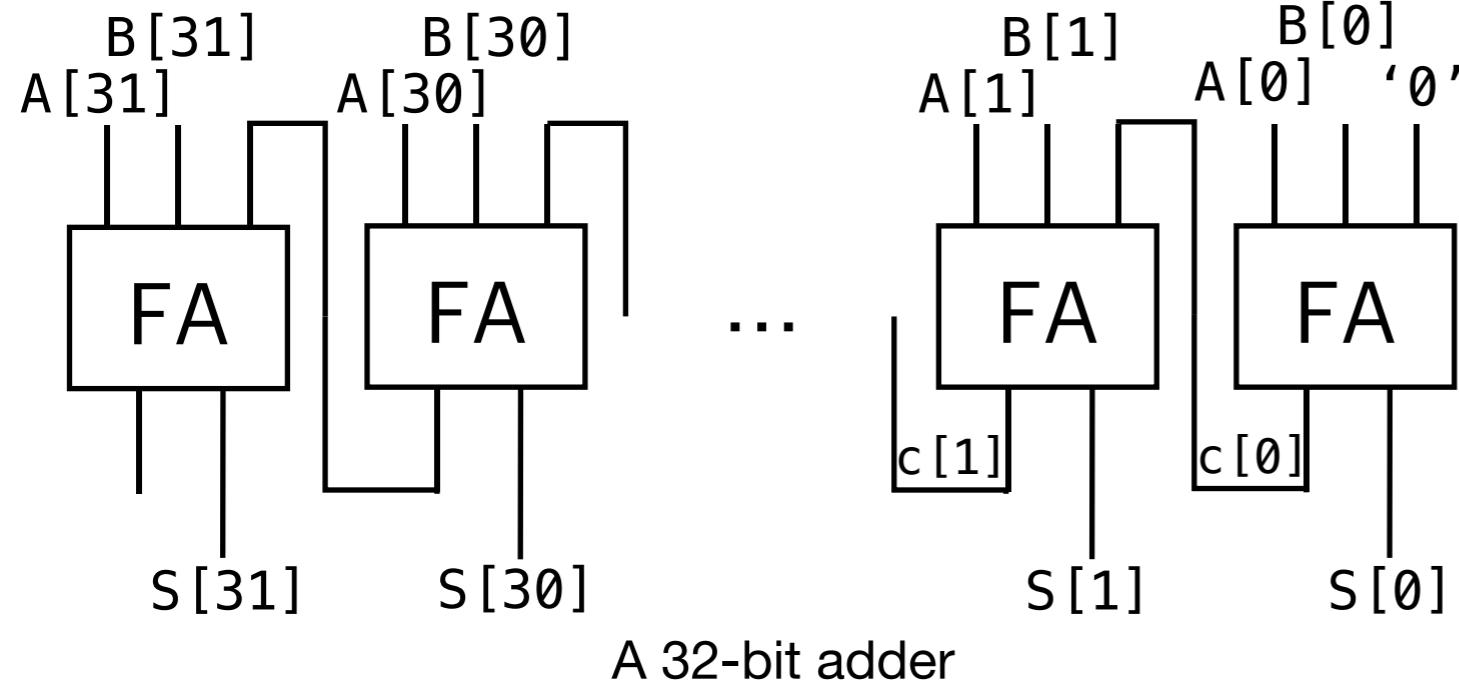
- A smart subtractor design
    - Recall that subtracting a number is equivalent to adding its negative version

# A smart subtractor design

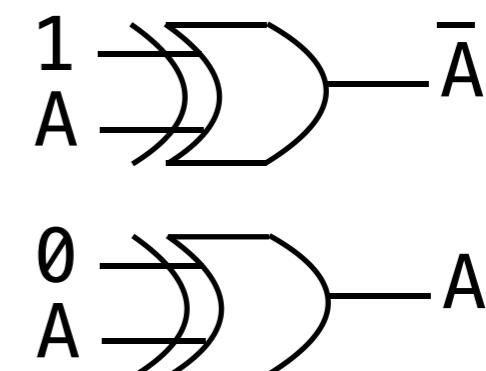
$$A - B = A + (-B) = A + \bar{B} + 1 \pmod{2^{N-1}}$$



A 32-bit subtractor

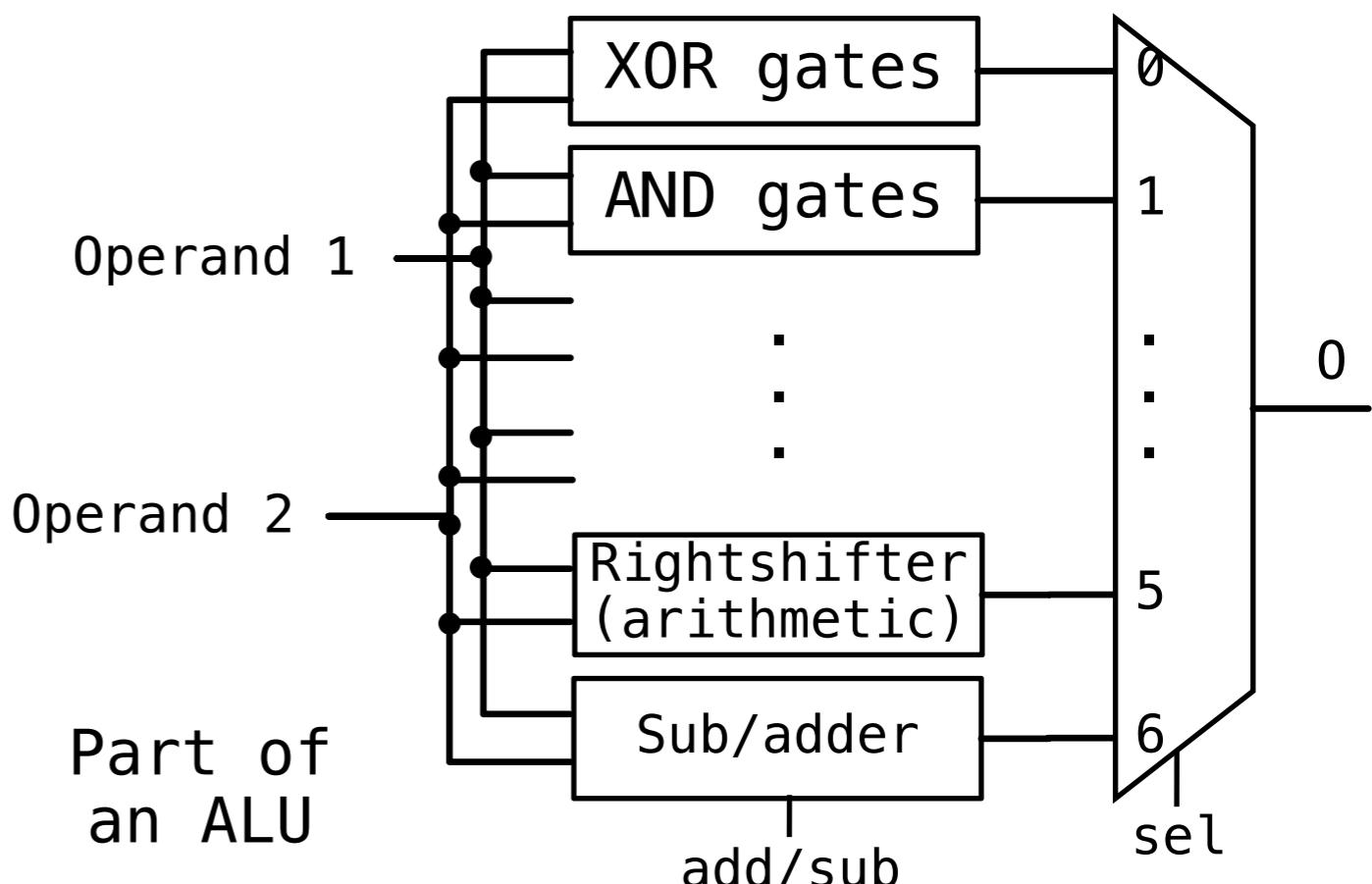
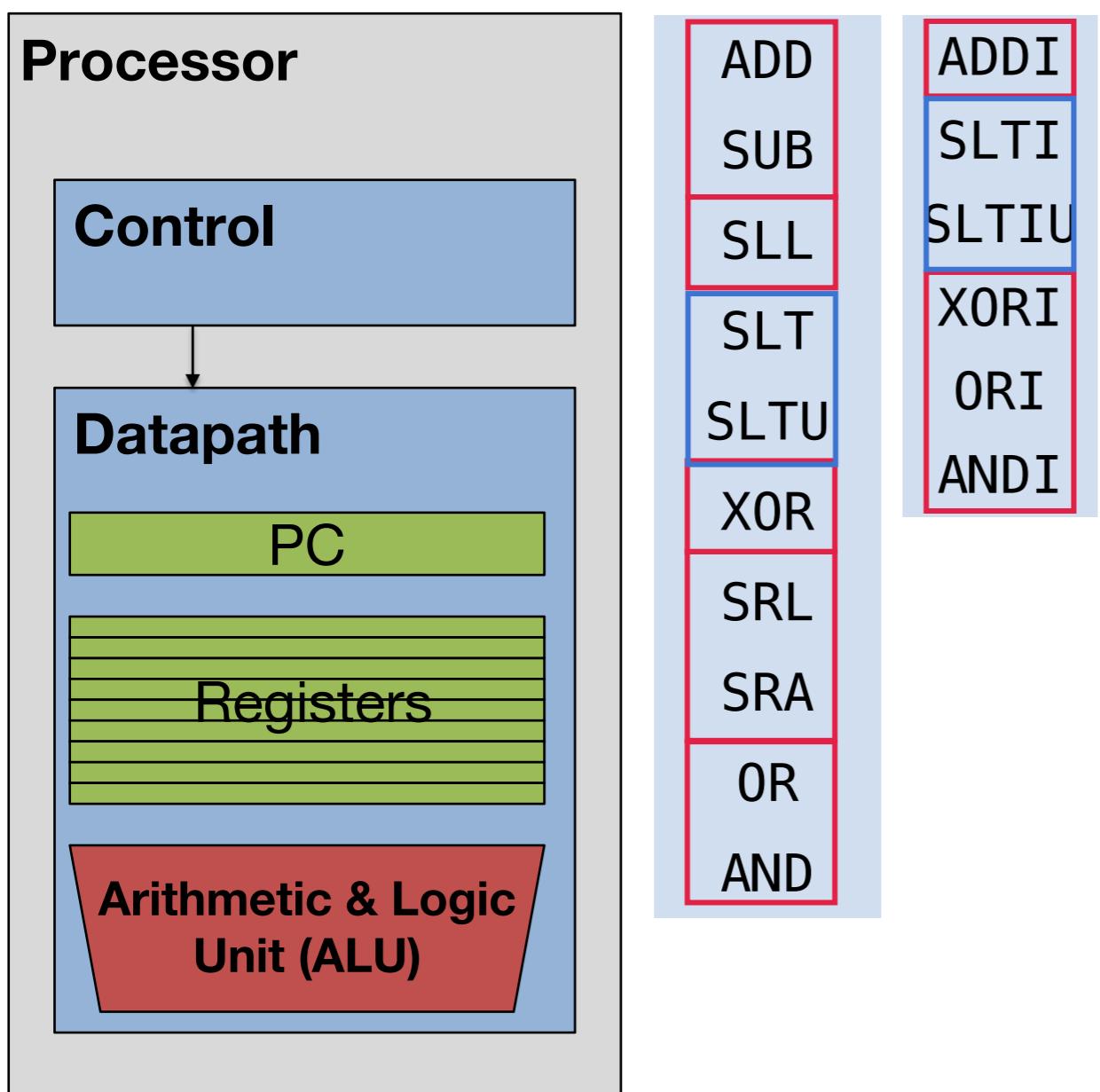


- Recall XOR gate



# Useful building blocks

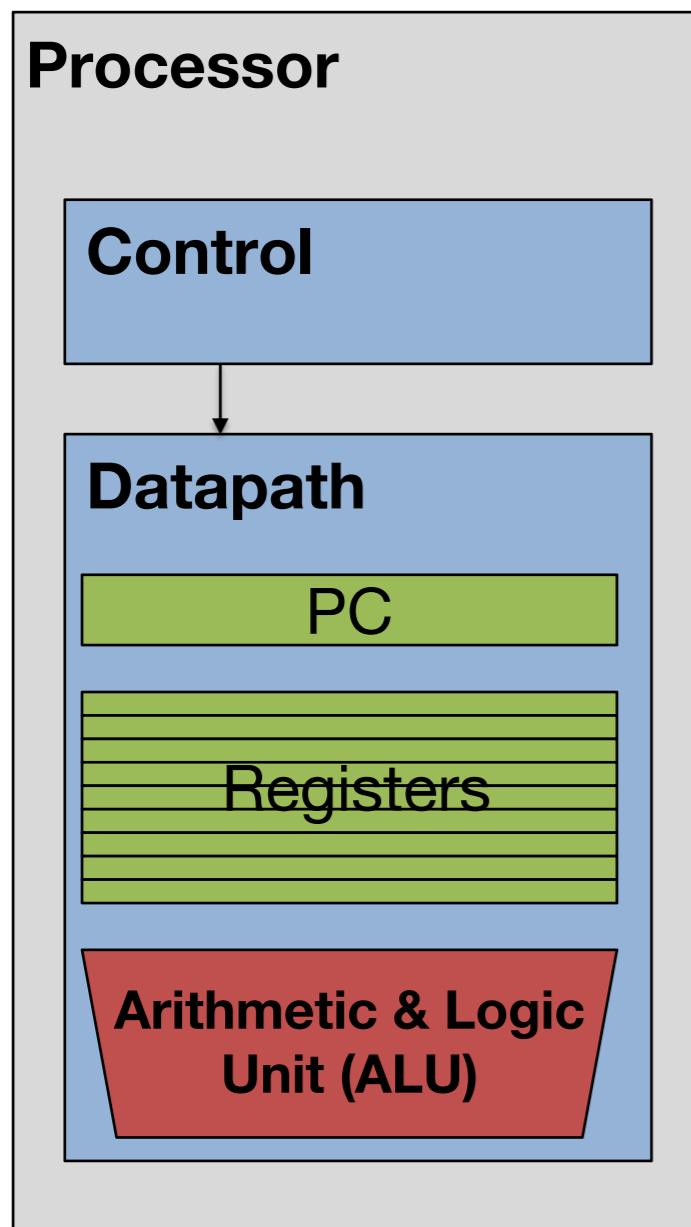
- An ALU should be able to execute all the arithmetic and logic operations



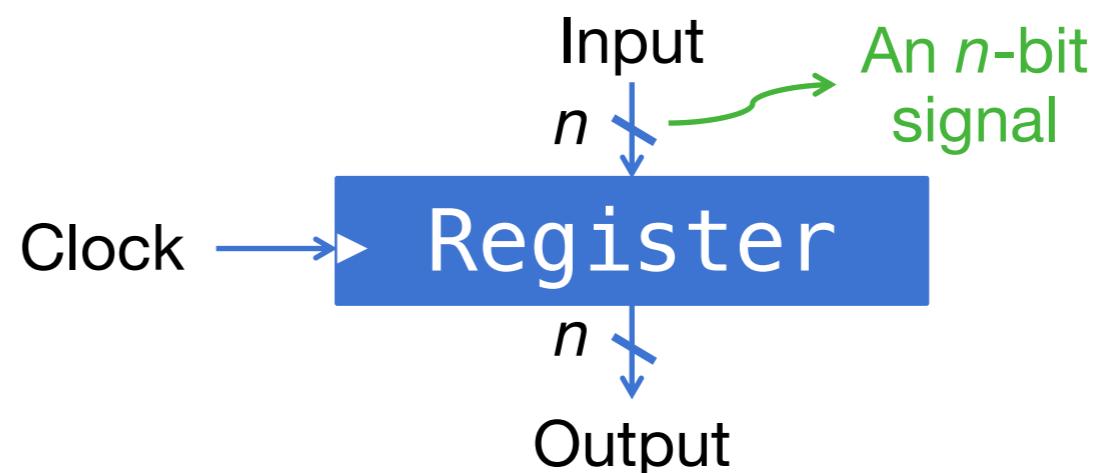
- ALU design that supports R/I-arithmetic and logic operations completed

# Useful building blocks-Register file

- The register file is the component that contains all the general purpose registers of the microprocessor
- A register file should provide data given the register numbers
- A register file should be able to change the stored value

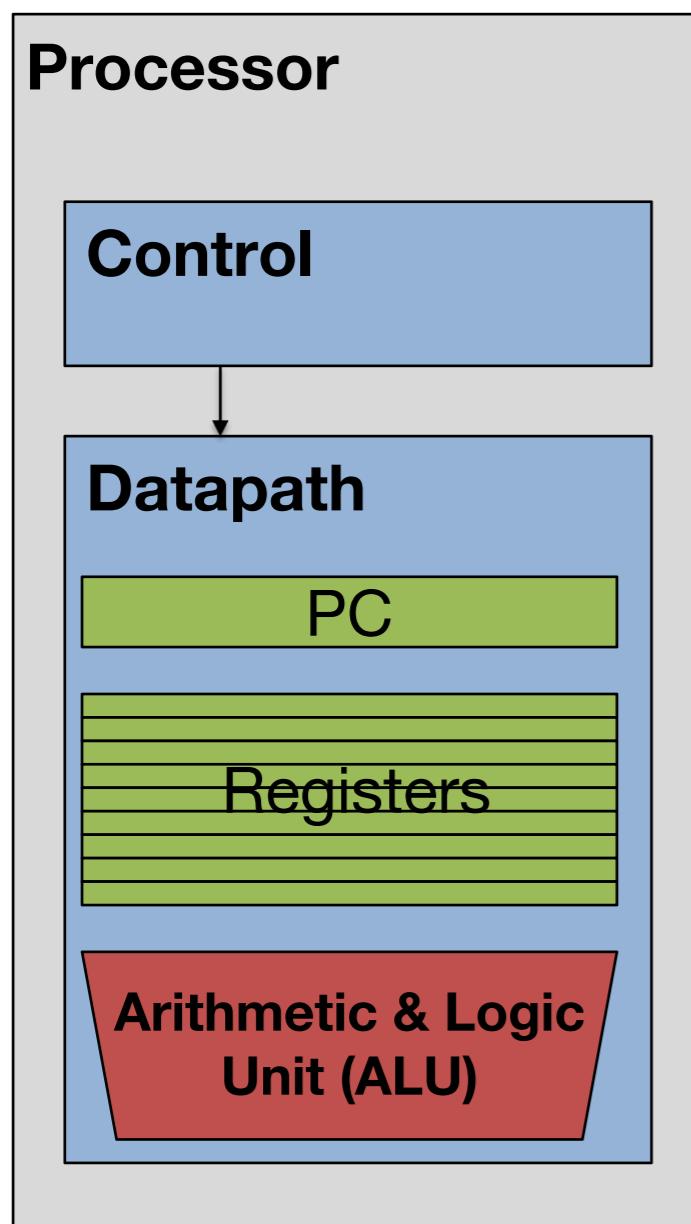


- Recall we have registers that store values

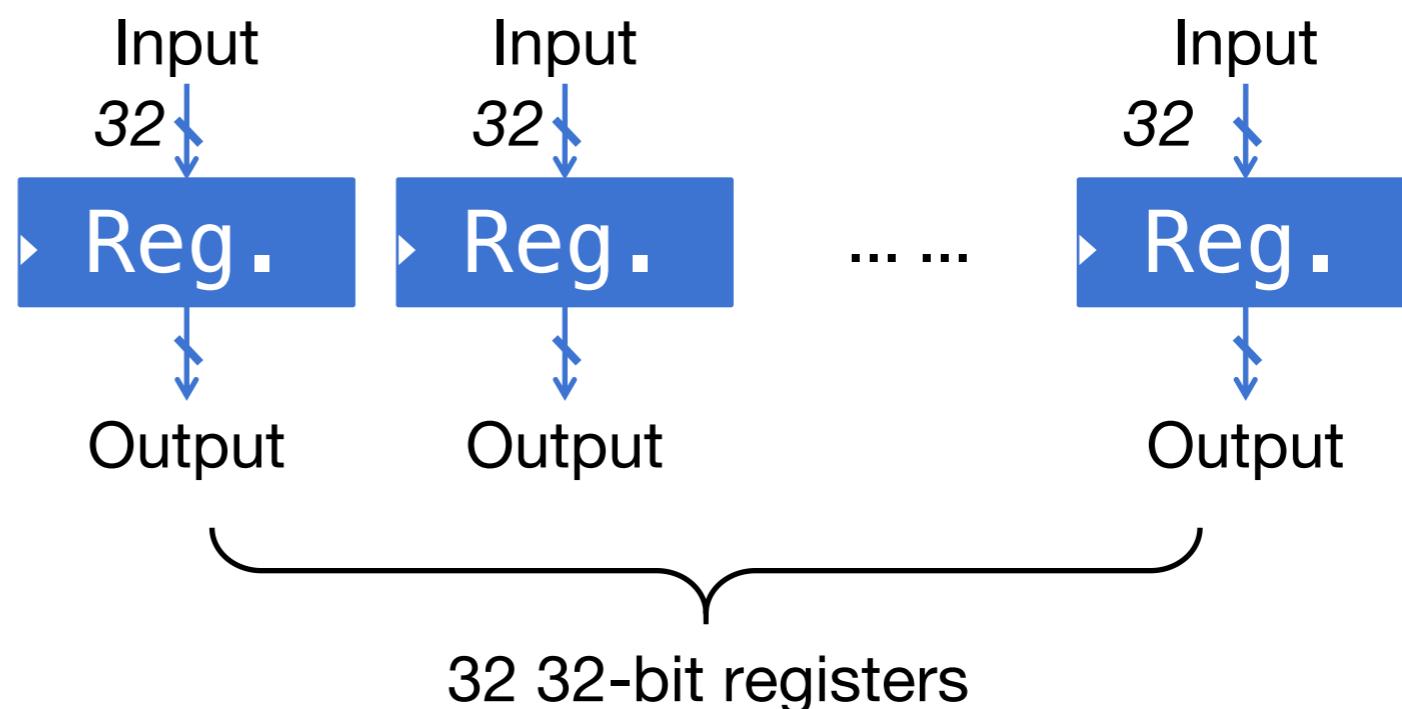


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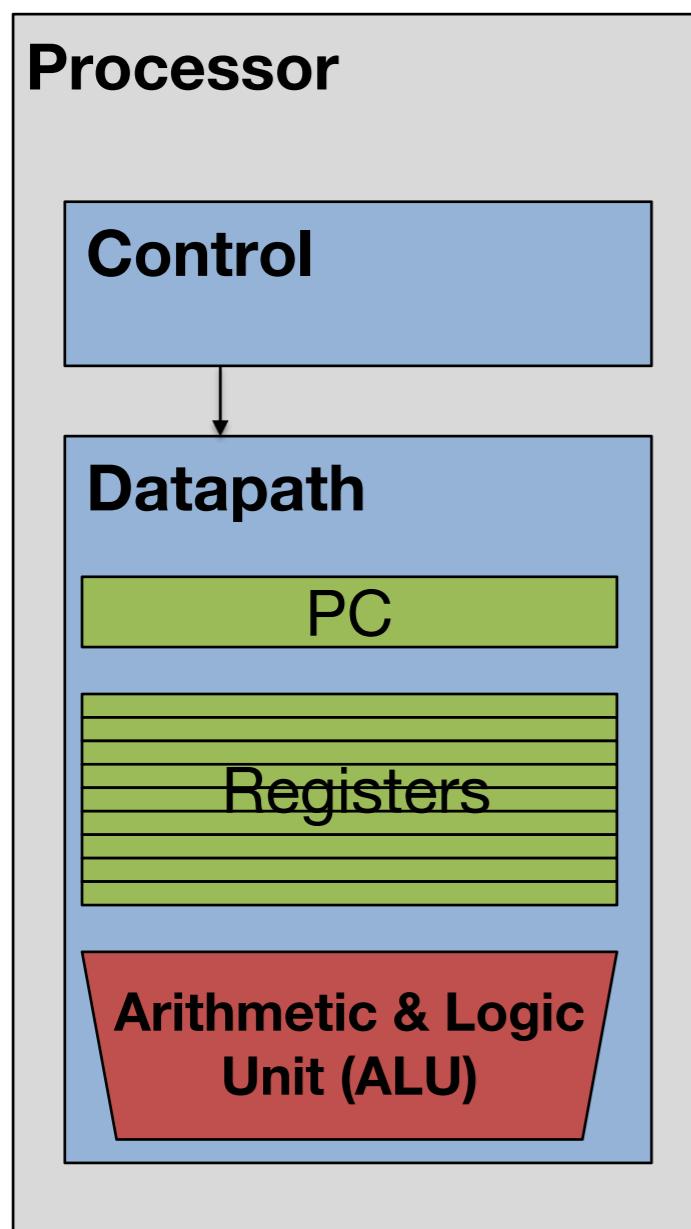
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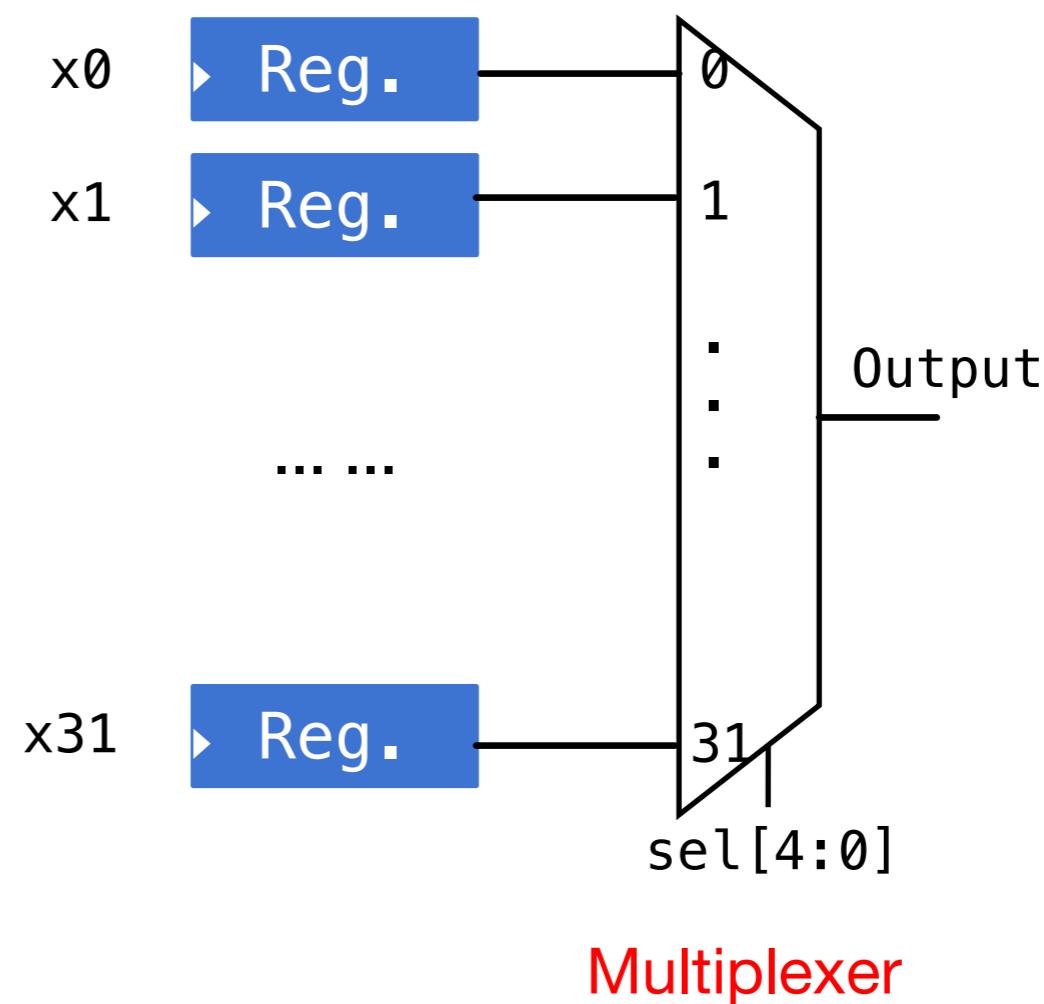
- How to select one to output? **Multiplexer**

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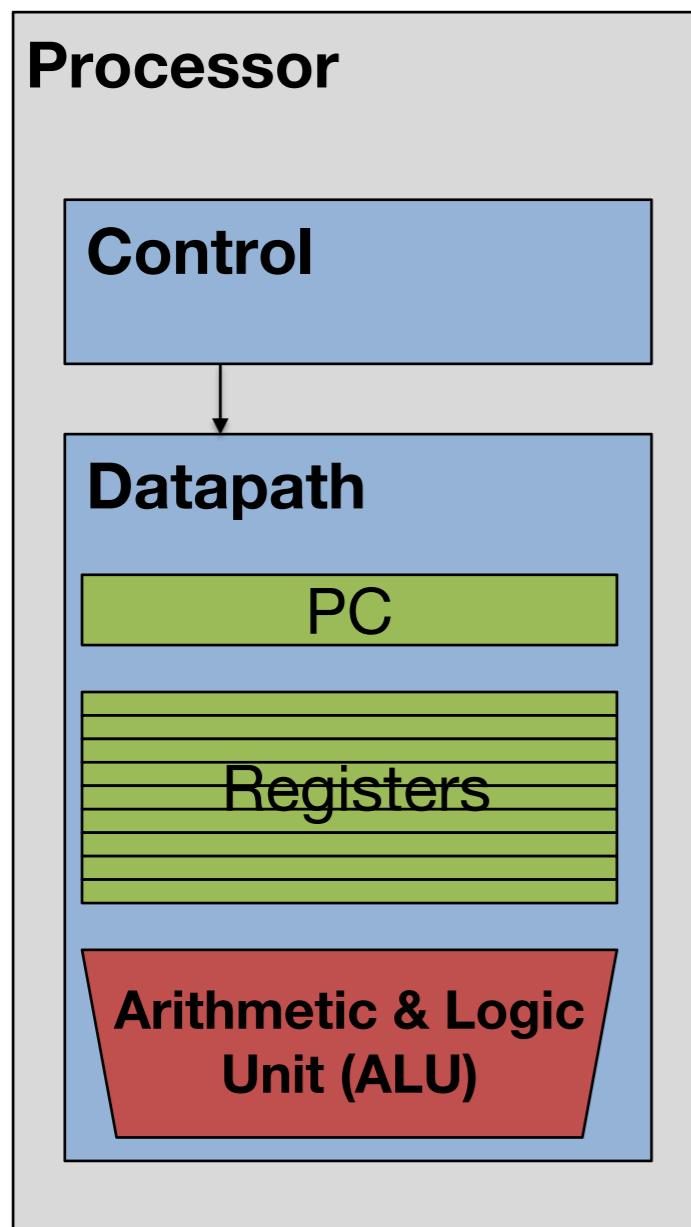


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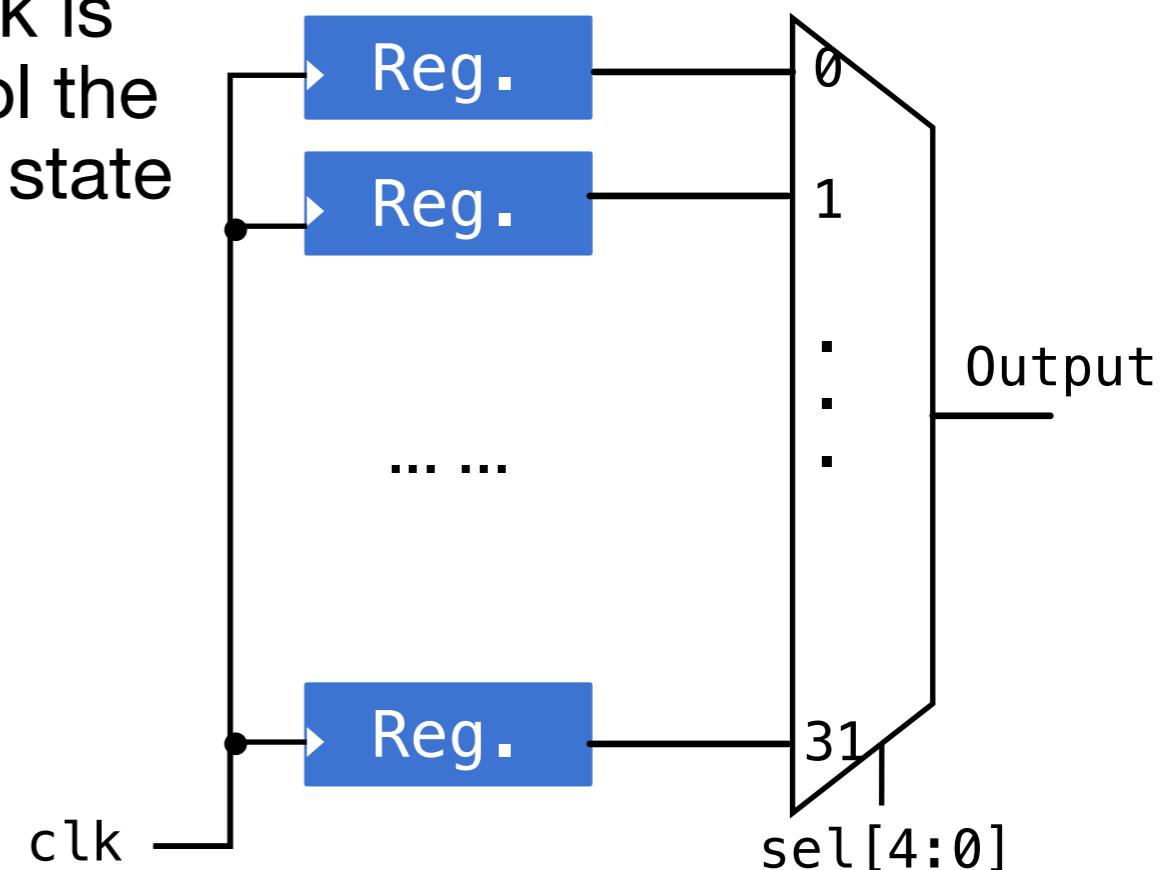


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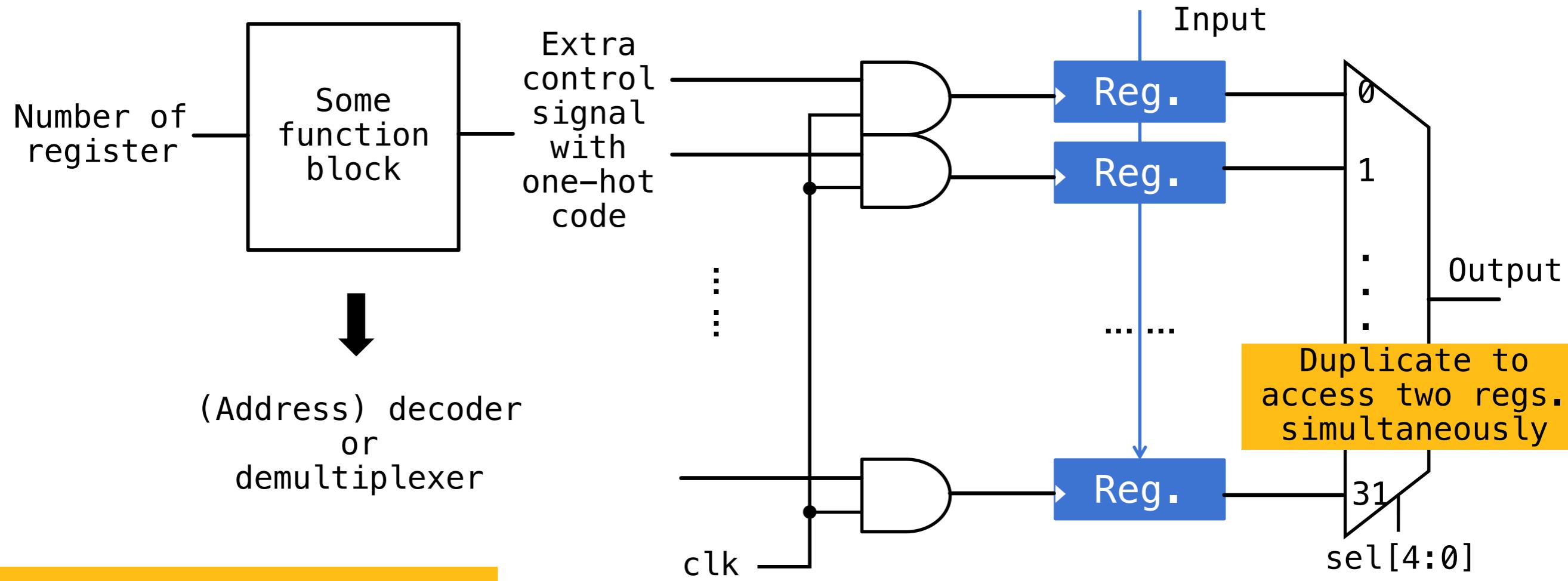
- How do we change values of a specific reg.?
- Recall that  $clk$  is used to control the change of the state



# Useful building blocks-Register file

- The register file is the component that contains all the general purpose registers of the microprocessor
  - A register file should provide data given the register numbers
  - A register file should be able to change the stored value

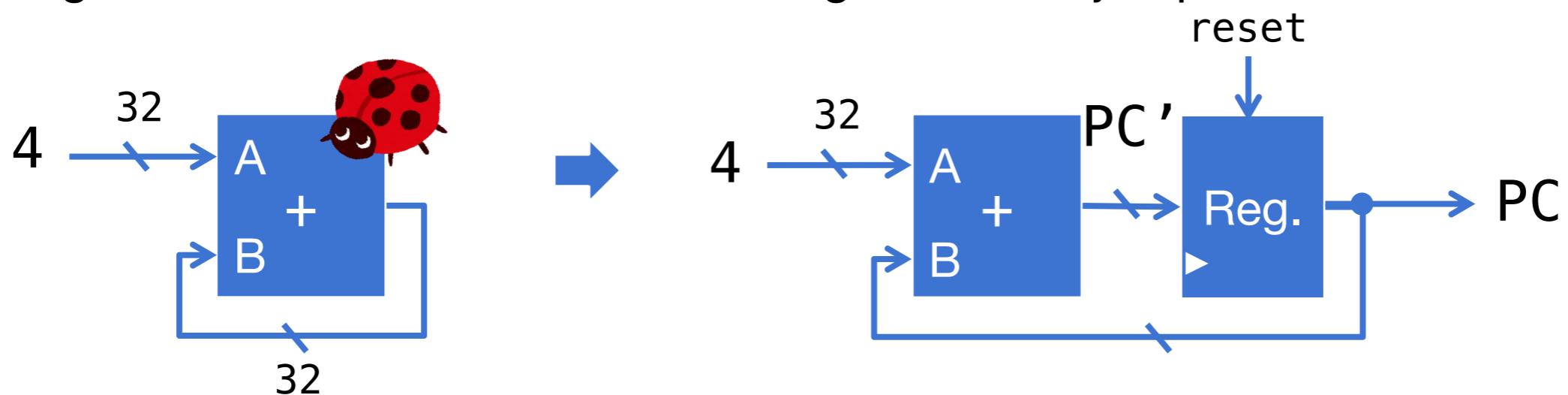
- How do we change values of a specific reg.?



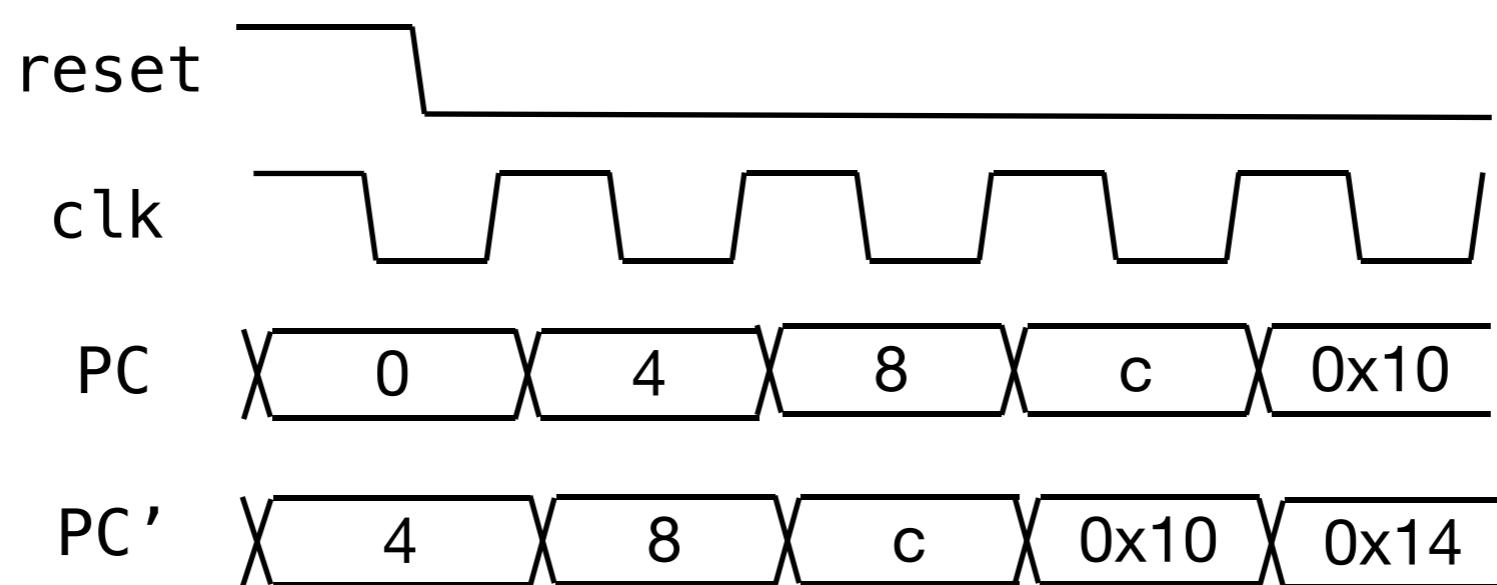
- Reg. file design completed

# We have covered PC register previously

- Synchronous digital circuit can have feedback, e.g., iterative accumulator
  - e.g.  $PC = PC + 4$  without considering branch or jump



- Timing diagram



# Useful building blocks-Memory

- Memory similar to register file except that the basic cell design is different
- Requires refresh for DRAM

